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John Neter

James K.

AUDITING RESEARCH MONOGRAPH

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BEHAVIOR OF MAJOR STATISTICAL ESTIMATORS IN SAMPLING ACCOUNTING POPULATIONS

AN EMPIRICAL STUDY

by John Neter
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AICPA
American Institute of Certified Public Accountants

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AUDITING RESEARCH MONOGRAPH

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AN EMPIRICAL STUDY

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Foreword

This is the second in the series of auditing research monographs published by the Institute in connection with its technical research program. The series was undertaken in the belief that research would be helpful in approaching and solving significant problems related to the audit function. This series supplements the research papers prepared directly for the Institute's Auditing Standards Executive Committee and provides a means to obtain public exposure and comment on research results.

New York, N. Y.
December 1975

PAUL ROSENFIELD, *Director*
Technical Research

Preface

As statistical sampling techniques have become more widely used by auditors, cooperative efforts have evolved between statisticians and auditors in the application of these techniques for appraising accounting populations. At the institutional level, this cooperation has taken place between the Statistical Sampling Subcommittee of the American Institute of Certified Public Accountants (AICPA) and the Committee on Statistical Sampling in Accounting of the American Statistical Association (ASA). It has been evidenced by the ASA committee's review and comment on volumes one through five of the AICPA Statistical Sampling Training series, and by contributions to ASA annual meetings by members of the AICPA subcommittee. Cooperation has also existed at the firm level, in that many CPA firms have engaged statisticians as consultants in the area of statistical sampling in auditing.

This study represents a continuation of the cooperative effort between auditors and statisticians. It began because of the lack of empirical evidence about the behavior of various statistical estimators when sampling actual accounting populations. Initially, the study was to be a relatively small exploratory one, with funding by Touche Ross & Co. and computer support by the University of Minnesota. Subsequently, the AICPA undertook to publish a text on statistical auditing, to be written by Donald M. Roberts of the University of Illinois. It became clear that broadening our research effort would accommodate the needs of this book. Accordingly, the project was expanded to its present state, with financial support by the AICPA and Touche Ross & Co. and with computer support by the University of Minnesota.

Despite the greater scope of the study made possible by AICPA support, the present effort still represents only an initial empirical investigation of a variety of important, but complex, problems. Rather than developing the study still further before reporting any results, we feel it is important to present the findings obtained thus far even if they do not answer all questions definitively. It is our hope that this cooperative study will not only have some immediate usefulness to both the auditing and statistical professions, but that it will encourage continuing cooperative research efforts in the uses of statistical methods in auditing. Further, it is our hope that the empirical study will foster theoretical investigations and that the interaction between empirical and theoretical studies will lead to better statistical tools and more effective applications of statistical sampling methods in auditing.

It is a pleasure to acknowledge our appreciation for assistance in this undertaking. The American Institute of Certified Public Accountants and Touche Ross & Co. provided the necessary financial support, and the University of Minnesota assisted by giving us needed computer time. Computer programming and related responsibilities were ably handled by C. Randall Byers, now at the University of Idaho, and by Izak Benbasat, now at the University of British Columbia. Helpful comments on the first draft of this monograph were received from John C. Broderick, Arthur Young & Company, Robert S. Kaplan, Carnegie-Mellon University, and Donald M. Roberts, University of Illinois and American Institute of Certified Public Accountants. We are most grateful to all these persons and organizations for their assistance.

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Part I

Introduction

1

Overview of Empirical Study

When the independent auditor audits the financial statements on a business, his objective is to gather sufficient, competent evidential matter to formulate an opinion on the fairness with which they present its financial position, results of operations, and changes in financial position in conformity with generally accepted accounting principles. This process, generally, involves making analytical or detailed tests.

- Analytical tests are broad, such as those concerned with changes in account balances between years, and are designed to determine the reasonableness of relationships.
- Detailed tests are specific, usually dealing with the individual items comprising a particular account balance, and have as their objective the determination of errors.

It is when making detailed tests that the auditor uses statistical sampling. These uses have been given impetus by the auditing environment and the power of the computer. Numerous articles on the subject have appeared in recent issues of accounting journals and, most significantly, statistical sampling is receiving strong recognition as an effective auditing technique by the American Institute of Certified Public Accountants.[1]

1. See Committee on Auditing Procedure, Statement on Auditing Standards No. 1, *Codification of Auditing Standards and Procedures,* (New York: American Institute of Certified Public Accountants, 1973), sections 320A and 320B.

When statistical sampling is employed in detailed tests, the auditor should use the sample data to do the following:

1. Estimate the total amount of errors in the population and determine how this magnitude affects the audit conclusions.
2. Examine the causes of errors found and determine their implications concerning (a) the client's system of internal control and (b) the appropriateness of the planned audit procedures.
3. Assess the effectiveness of the audit procedures performed in that test.[2]

Historically, the auditor has turned to sample survey techniques when using statistical sampling for these purposes. The AICPA has published a series of teaching booklets that include four methods of sample evaluation: attribute estimation, mean-per-unit estimation, difference estimation, and ratio estimation.[3] More recently some auditors have developed procedures using a combination of these techniques,[4] which have been called combined attributes-variables (CAV) procedures.[5] Also, some auditors are now stressing the use of sample results in a testing-of-hypotheses framework rather than in an estimation framework.[6]

Problems When Sampling Accounting Populations

When the auditor samples accounting populations, he encounters two key sets of problems. One is connected with low error rates in accounting populations. The other pertains to the effectiveness of sampling procedures designed for either low or high error rates when the actual error rate is not of the anticipated magnitude.

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2. See James K. Loebbecke and John Neter, "Statistical Sampling in Confirming Receivables," *Journal of Accountancy,* June 1973, pp. 44–50.
3. AICPA, *An Auditor's Approach to Statistical Sampling,* vols. 1–6 (New York: American Institute of Certified Public Accountants, 1973–1974).
4. Giles R. Meikle, *Statistical Sampling in an Audit Context* (Toronto: The Canadian Institute of Chartered Accountants, 1972); Rod Anderson and A. D. Teitlebaum, "Dollar-unit Sampling," *Canadian Chartered Accountant,* April 1973, pp. 30–39.
5. James L. Goodfellow, James K. Loebbecke, and John Neter, "Some Perspectives on CAV Sampling Plans," Part I, *CA Magazine,* October 1974, pp. 23–30; Part II, *CA Magazine,* November 1974, pp. 46–53.
6. See, for example, Robert K. Elliott and John R. Rogers, "Relating Statistical Sampling to Audit Objectives," *Journal of Accountancy,* July 1972, pp. 46–55.

Low Error Rates. Auditors are frequently faced with populations containing low error rates. As it has been noted for some time, this creates problems when the commonly used ratio and difference estimators are employed.[7] In particular, the sample frequently will contain no errors in this situation and the estimated standard error will then be zero, a meaningless result for the auditor. Even if each sample contains a few errors, the actual confidence coefficient may be substantially different from the specified one when the normal distribution is used in the construction of the confidence interval and the actual distribution is far from normal.

Robert S. Kaplan has investigated these evaluation problems by simulation studies using hypothetical data; his results suggest that these problems are serious.[8]

Effectiveness of Procedures When the Error Rate Is of a Different Magnitude. The evaluation problems associated with low error rate populations are further compounded because the auditor often does not know what rate of error to expect until *after* his test is performed. Since some of the most common types of auditing tests are difficult to "reopen," this creates further serious problems for the auditor. In effect, the auditor must do either of the following:

1. Plan a sample design and method of evaluation which is not only suitable for the anticipated population error rate, but which would also be effective if the error rate were of a different magnitude.
2. Plan for a fallback method of sample evaluation in case his anticipations about the population error rate are sufficiently off the mark so that his primary method of evaluation would be inappropriate.

Purposes of This Study

Not only is there a scarcity of empirical research on the statistical problems related to low error rates in accounting populations, but there is even a great lack of systematic information about the characteristics of accounting populations and of error patterns found in them. This project constitutes a first step in empirically studying the sampling behavior of major statistical estimators for actual accounting populations and error patterns.

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7. See, for example, K. W. Stringer, "Practical Aspects of Statistical Sampling in Auditing," in *Proceedings of the Business and Economic Statistics Section, 1963* (Washington, D.C.: American Statistical Association, December 1963).
8. Robert S. Kaplan, "Statistical Sampling in Auditing with Auxiliary Information Estimators," *Journal of Accounting Research,* Autumn 1973, pp. 238–258.

The following three major questions were posed for this empirical study:

1. Can effective use of sample design permit moderate sample sizes and also lead to reliable results in the presence of low error rates?
2. If the sampling plan is designed for low error rates, how effective will it be in the presence of high error rates?
3. Is it possible to determine when the population error rate and sample size are large enough for difference and ratio estimators to be reliable and effective?

Related questions were also investigated as follows:

1. Is poststratification an effective fallback technique if simple random sampling of audit units is employed with the expectation that ratio or difference estimators can be utilized but a low error rate is encountered?
2. Is a combination of mean-per-unit estimation and either ratio or difference estimation with simple random sampling of audit units an effective fallback procedure if ratio or difference estimation is ineffective?
3. When stratification of audit units is based on their book values, what is the effect of the number of strata utilized if substantial error rates are encountered?
4. When dollar-unit sampling is employed, is mean-per-unit estimation an effective fallback procedure if combined attributes-variables bounds are ineffective when substantial error rates are present?

General Approach of This Study

For each of a number of accounting populations, a variety of sample designs, selection procedures, and evaluation methods with a potential use by auditors were considered. The approach was to select a large number of samples from each population for each sampling plan studied and examine the estimates obtained from these samples for relevant behavior characteristics.

Populations Studied. Four accounting populations were used in this study. For each of them, audit data were available from a sample of the audit units to provide information on the nature of the error pattern. The four populations can be characterized as follows:

Population 1. Accounts receivable of a freight company. The actual error rate is high, and the errors tend to be balanced between overstatements and understatements.

Population 2. Inventory of a medium-size manufacturer. The actual error rate is very high. Both overstatement and understatement errors are present, with understatement errors outweighing the overstatement errors.

Population 3. Accounts receivable of a medium-size manufacturer. The actual error rate is moderate, with all errors being overstatements.

Population 4. Accounts receivable of a large manufacturer. The actual error rate is moderate, with all errors being overstatements.

Thus, the four accounting populations come from two areas often audited through statistical sampling techniques and reflect some of the contrasting characteristics of error patterns found in auditing.

Creation of Study Populations. From each of the four populations, several study populations with a variety of error rates were created. This was done by utilizing the error pattern actually found to assign errors at random to the audit units in the population to achieve the specified error rate. Thus, it was possible to study the behavior of statistical estimators for different error rates in the same population based on the same error pattern.

The error rates in the study populations include the low rates of .5% and 1%, the moderate rate of 5%, and the high rates of 10% and 30% (for one population, the highest error rate is 70%).

Sample Selection Procedures and Estimators. The following sample selection procedures and statistical estimators were examined:

1. Simple random sampling of audit units
   a. mean-per-unit estimator
   b. difference estimator
   c. ratio estimator
   d. combined mean-per-unit and difference estimator
   e. combined mean-per-unit and ratio estimator
   f. poststratified mean-per-unit estimator
2. Stratified random sampling of audit units
   a. mean-per-unit estimator
   b. difference estimator
   c. ratio estimator
3. Simple random sampling of dollar units
   a. combined attributes-variables bound
   b. mean-per-unit estimator

Sampling Experiments. Each sampling procedure was utilized for all study populations for each population, except in a few cases where it was deemed adequate that only some of the error rate study populations be considered.

In all cases, two sample sizes were employed: $n = 100$ and $n = 200$. The sample size of 100 was chosen as typically being a minimum sample size for detailed substantive audit tests; the sample size of 200 was selected as representing a moderate sample size.

For each sample selection method and sample size, 600 samples were selected from the study population. For each such sample, all of the estimates used with the given sample selection procedure were then calculated. Thus, for each of the 600 simple random samples of audit units, the mean-per-unit, difference, ratio, combined mean-per-unit and difference, combined mean-per-unit and ratio, and poststratified mean-per-unit estimates were obtained.[9] These results were then analyzed for relevant behavior characteristics. The choice of 600 repetitions was made to yield reasonably precise estimates of the actual confidence coefficient obtained with any particular procedure.[10]

Throughout the study (with one exception), the total audit value of the items in the population was estimated. All of the results are, however, directly applicable to estimates of the total error in the population obtained by subtracting the estimated audit value from the total book value. The reason is that the two estimates differ by a constant quantity, which does not affect the standard error of the estimator and other relevant characteristics.

9. Comparisons between different estimators for a given sample selection method, therefore, involve only the effects of the different estimators and not of any differences between samples.
10. The estimated standard error of the proportion of correct confidence intervals in 600 replications is as follows, for selected levels of the proportion of correct intervals:

Proportion of correct intervals	Estimated standard error
80%	.016
90%	.012
95%	.009
99%	.004

Thus, the choice of 600 replications gives relatively precise information about the actual confidence level in the range of practical interest.

Analysis of Sampling Experiments

The analysis of the sampling experiments focused on the following two major characteristics of the statistical procedures:

1. How precise is the estimator?
2. How reliable is the nominal confidence coefficient based on use of the normal distribution?

The findings of the study strongly suggest that no one statistical procedure is optimal under all circumstances with regard to precision and reliability of the nominal confidence coefficient. However, at least one of the statistical procedures was reasonably effective for each of the populations and error patterns considered in this study.

Outline of Presentation

This chapter has given an overview of the entire research project. Chapter two describes each of the four populations studied, the error patterns found, and the characteristics of the study populations created with different error rates. Chapter two also explains the summary statistics presented in the Appendix tables as well as some other matters related to the analysis of the results.

Part two presents the results of the sampling experiments based on simple random sampling of audit units, devoting a chapter to each of the estimators studied for that type of sample selection. Parts three and four, devoted to stratified random sampling of audit units and simple random sampling of dollar units respectively, are similarly organized.

The reader who is not concerned with the detailed statistical findings but is mainly interested in the major results and in their implications for auditors may wish to omit Parts two–four and turn directly to Part five, which contains a summary and discussion of the findings.

2

Populations Studied and Methods of Analysis

This chapter describes each of the four accounting populations utilized in the study by the following three main elements:

1. The chief characteristics of the book values.
2. The nature of the error pattern.
3. The chief characteristics of the audit values for the study populations generated with different error rates.

In addition, this chapter contains a discussion of the methods of analysis employed in this study.

Population 1

Population 1 consists of 8,309 accounts receivable of a freight company and is a random subset of this company's very large population of accounts receivable.

Characteristics of Book Values. Table 2.1 (p. 12) contains a frequency distribution of the book amounts of the 8,309 accounts receivable as well as some of the major characteristics of these book values.[1] The mean, standard deviation, skewness, and kurtosis measures are defined in analogous fashion to those in formulas (2.2) through (2.5), pp. 29 and 30. Figure 2.1,

Table 2.1
Frequency Distribution and Major Characteristics of Population 1 Book Values

Book Amount		Number of Accounts
0– 13.50		2,039
13.51– 22.50		2,455
22.51– 36.00		1,867
36.01– 63.00		852
63.01– 105.00		494
105.01– 195.00		335
195.01– 345.00		136
345.01– 675.00		79
675.01– 945.00		24
945.01–1,545.00		16
1,545.01–6,945.00		12
	Total	8,309

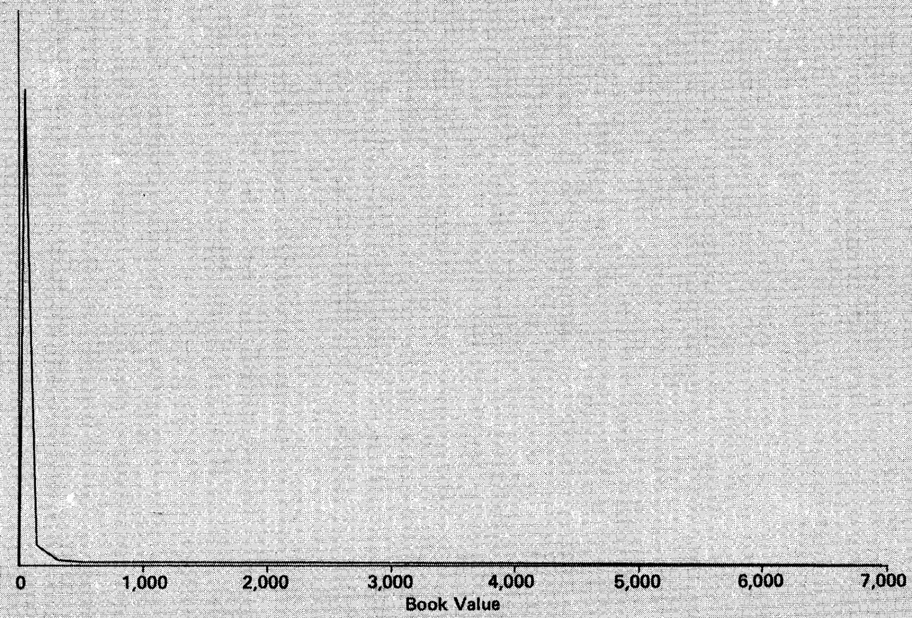
Total book value	$379,131.00
Mean	$ 45.63
Standard deviation	$ 132.61
Skewness	22.0
Kurtosis	906.4
Maximum	$ 6,869.70
Minimum	$ .50

opposite, shows the frequency distribution of the book amounts in the form of a frequency polygon.

The high degree of positive skewness—with most of the account balances falling under $100 but some amounting to thousands of dollars—is readily apparent. The measure of skewness is +22. Also, the population standard deviation is almost three times as large as the population mean.

1. All book amounts are greater than zero for population 1, as well as for the other three populations studied. Items having a zero or a negative book amount were eliminated on the assumption that if these are not of negligible importance, the auditor will wish to audit them separately.

Figure 2.1 Frequency Polygon of Population 1 Book Values



Under such conditions of high skewness, the auditor would very likely study the largest accounts on a 100% basis and sample only the remaining accounts. It was decided, however, to study all sampling procedures from the actual highly skewed population, without initially stratifying out the largest items. In this way, the characteristics of the various statistical procedures could be examined under the most adverse conditions for at least one of the four populations included in this study.

Characteristics of Errors. An audit of 555 accounts from the population disclosed 159 accounts in error, or an error rate of 28.6%.

An error is defined in this study as the difference between the book value and the audit value:[2]

$$E = Y - X \tag{2.1}$$

where E denotes the error amount, Y the book value and X the audit value. Thus, an overstatement in the book value implies a positive error and an understatement a negative error.

Both positive and negative errors are present in the 555 accounts and tend to balance out, with a mean error of \$.22. The largest understatement error is −\$35.15, and the largest overstatement error is \$46.17. Thus, the error rate is high but the absolute amounts of errors are not very large. Indeed the mean absolute error is only \$3.89.

There is a slight positive relation between the amount of the book value and the expected amount of error, but this is entirely due to the three largest accounts with errors. If these three accounts are omitted, there is no significant tendency for the expected amount of error to vary with the book value.

In contrast, the absolute amount of error does tend to increase with the book value of the account. For example, the mean absolute error for accounts with book values under \$10 is \$.88, while that for accounts with book values over \$400 is \$33.

There is no strong evidence that the error rate for large accounts is substantially different than that for small accounts. This conclusion is supported by the fact that the mean book value of the audited accounts with errors differs by only 2% from the mean book value of audited accounts with no errors.

Creation of Study Populations. The 159 errors found in the audit of 555 accounts receivable were used as a pool for creating study populations with

2. The definition of error follows standard statistical usage. In some accounting literature an error is defined in reverse fashion, namely, as audit value minus book value.

different error rates. Specifically, populations with error percentages of .5, 1, 5, 10, and 30 were created. Since the absolute errors tend to increase with book value, the 159 errors were divided into five error pools, according to the book value of the account. Thus, one error pool pertained to accounts with book values under $10, another to accounts with book values between $10 and $24.99, and so on. In each error pool, the largest understatement and overstatement errors were each increased by 20% to partially allow for the fact that the audit sample may not have disclosed the largest errors in the population. All other errors in the error pool remained unchanged. Table 2.2, below, shows the range of the error amounts and the mean error amount for each error pool.

Table 2.2
Range and Mean of Error Amounts in Five
Error Pools for Population 1

		Error Range		
Error Pool	Book Amount	Minimum*	Maximum*	Mean
1	0- 9.99	− 3.60	.84	− .71
2	10.00- 24.99	−33.60	10.09	−1.42
3	25.00- 99.99	−12.38	20.88	1.29
4	100.00–399.99	−31.26	14.76	− .96
5	400.00 or more	−42.18	55.40	9.93

*The extremes have been increased by 20% over the actual extreme error amounts.

The procedure to create the study population with a 30% error rate was then as follows:

1. From the 8,309 accounts in the population, 2,493 accounts were selected at random (the number being determined according to the desired 30% error rate). (This procedure is consistent with the lack of strong evidence of differential error rates according to the book amount.)
2. For each selected account, the book value was determined and an error amount chosen at random with replacement from the appropriate error pool. (If the error amount led to a zero or negative audit amount, it was replaced by another error amount selected at random because none of the errors found in the audit study led to a zero or negative audit amount.)

The other study populations were developed by randomly eliminating errors in the 30% error rate population.

Characteristics of Study Populations. Table 2.3, below, presents the major characteristics of the audit values of each of the five study populations. In view of the balanced nature of the errors and their small absolute magnitudes, the five study populations have almost identical characteristics, except for the error rate.

Table 2.3
Major Characteristics of Audit Values in Study Populations for Population 1

	Population Error Percentage				
Characteristic	.5	1	5	10	30
Total audit value	$379,181.00	$379,264.00	$379,090.00	$379,547.00	$379,921.00
Mean	$ 45.64	$ 45.64	$ 45.62	$ 45.68	$ 45.72
Standard deviation	$ 132.61	$ 132.67	$ 132.53	$ 132.58	$ 132.56
Skewness	22.0	22.0	22.1	22.0	21.9
Kurtosis	906.4	904.8	908.5	899.2	899.2

Population 1M

Population 1M is a modified version of population 1. It represents that segment of the total population that an auditor will likely subject to sampling. As Table 2.1 indicates, there are a relatively few very large accounts in population 1. Specifically, there are 27 accounts with book balances over $950 representing about 12% of the total book value. An auditor is likely to examine these large accounts on a 100% basis. Hence, population 1M was defined to consist of all accounts with book balances of $950 or less.

Table 2.4, below, presents the major characteristics of the book amounts for population 1M. It is evident that the skewness of this population is much

Table 2.4
Major Characteristics of Population 1M Book Values

Total book value	$334,212.00
Mean	$ 40.35
Standard deviation	$ 72.32
Skewness	6.1
Kurtosis	48.3
Maximum	$ 948.28
Minimum	$ .50

less than that of population 1, although population 1M is still substantially skewed to the right.

Three study populations, corresponding to 1, 5, and 10% error rates, were created for population 1M by deleting accounts with book balances of over $950 from the corresponding study populations for population 1. Table 2.5, below, presents the major characteristics of the audit values for the three study populations. Again, all three study populations have very similar characteristics and differ primarily in their error rates.

Table 2.5
Major Characteristics of Audit Values in Study Populations for Population 1M

Characteristic	Population Error Percentage 1	5	10
Total audit value	$334,303.00	$334,162.00	$334,618.00
Mean	$ 40.36	$ 40.35	$ 40.40
Standard deviation	$ 72.36	$ 72.17	$ 72.42
Skewness	6.1	6.1	6.1
Kurtosis	48.4	48.4	48.8

Population 2

Population 2 consists of 5,482 inventory items—a random subset of the inventory items in the finished goods and component parts segments of a medium-size manufacturer's inventory. This inventory is kept on a perpetual inventory system with periodic test counts. A small number of inventory items with book values over $10,000 (up to several hundred thousand) are not included in population 2 because these would probably be examined on a 100% basis by an auditor.

Characteristics of Book Values. Table 2.6 (p. 18) contains a frequency distribution of the book amounts of the 5,482 inventory items as well as the major characteristics of these book values. Figure 2.2 (p. 19) presents a frequency polygon of the distribution of book amounts. Population 2, like population 1, is skewed to the right, but much less so. In fact, the skewness is not as severe as in population 1M, where the largest elements were also eliminated.

Characteristics of Errors. An audit of 217 inventory items was conducted and it disclosed 155 errors, or an error rate of 71%. Both errors of understatement and overstatement are present among the inventory items with book values of $10,000 or less. Understatement errors tend to dominate, the mean error being −$26. The absolute magnitude of these errors is relatively quite large, the mean absolute error amount being $179. The largest understatement error for inventory items with book values of $10,000 or less is −$2,163 and the largest overstatement error is $1,450.

Similar to the error pattern for population 1, there is no strong evidence of error rate differences by book amount for population 2. If such differences are present, they are likely to be small. Also, there is no clear evidence that the expected error amount varies with the book amount for inventory items of $10,000 or less. On the other hand, the absolute error clearly increases with larger book amounts. The mean absolute error is $17 for inventory items with book amounts under $100 but is $381 for items with book amounts between $5,000 and $10,000.

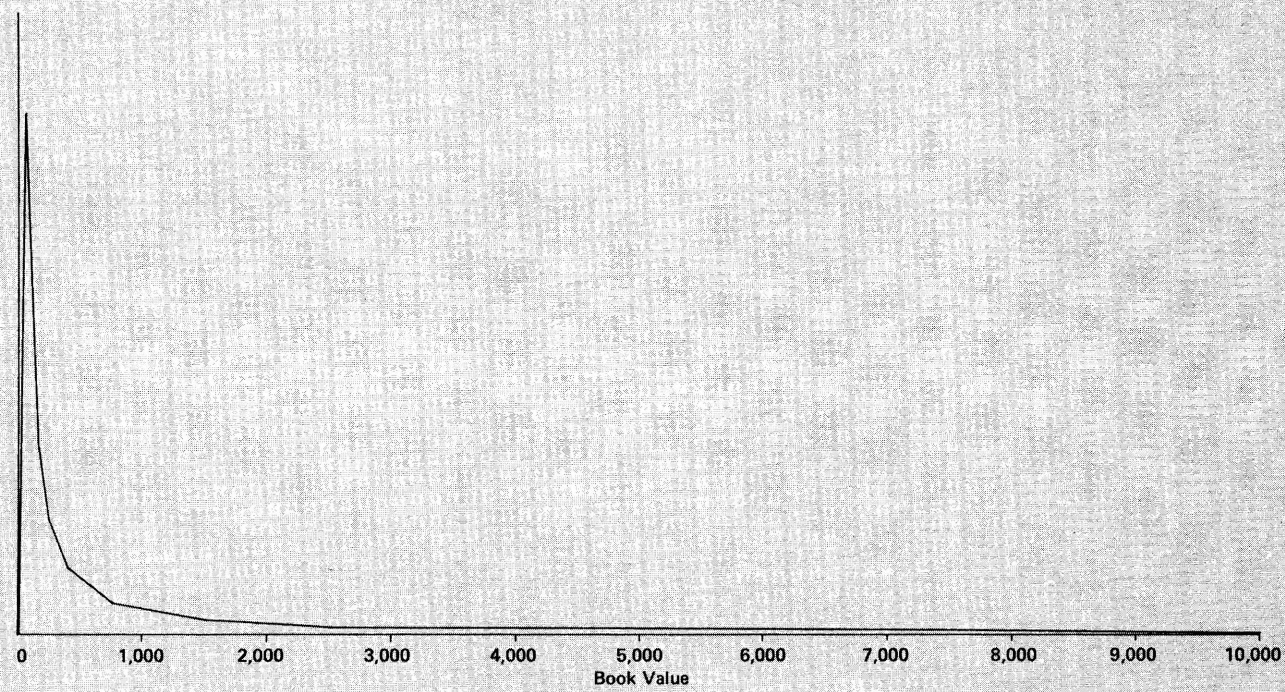
Creation of Study Populations. The 133 errors in inventory items with book amounts of $10,000 or less found in the audit sample were formed into

Table 2.6
Frequency Distribution and Major Characteristics
of Population 2 Book Values

Book Amount		Number of Items
0– 50.00		1,419
50.01– 100.00		676
100.01– 200.00		745
200.01– 300.00		457
300.01– 500.00		538
500.01– 1,000.00		636
1,000.01– 2,000.00		551
2,000.01– 3,000.00		195
3,000.01– 5,000.00		175
5,000.01–10,000.00		90
	Total	5,482

Total book value	$3,486,530.00
Mean	$ 636.00
Standard deviation	$ 1,155.99
Skewness	3.5
Kurtosis	15.2
Maximum	$ 9,989.00
Minimum	$ 1.00

Figure 2.2 Frequency Polygon of Population 2 Book Values



five error pools according to book amount to recognize that the absolute amount of error varies with the book amount. Table 2.7, below, shows the definitions of the five error pools and the range and mean of the error amounts for each of them. Again, the largest understatement and overstatement error amounts in an error pool have each been increased by 20% to partially allow for the possible presence of larger errors in the population than those found in the audit sample.

Five study populations were created, with error percentages of .5, 1, 5, 10, and 70, respectively. It was assumed, in accordance with the audit evidence, that the probability of an inventory item having an error was constant, regardless of book amount. The procedure to create the study population with a 70% error rate was as follows:

1. From the 5,482 inventory items in the population, 3,837 inventory items were selected at random.
2. For each selected inventory item, the book value was determined and an error amount was chosen at random with replacement from the appropriate error pool. (If the error amount led to an audit value less than $0, the error amount was adjusted to yield an audit value of $0. There were instances in the audit sample where the audit value was $0, but none with a negative balance.)

The other study populations were created by randomly eliminating errors in the 70% error rate population.

Characteristics of Study Populations. Table 2.8, opposite, presents the major characteristics of the audit values of the five study populations. The study populations are very similar in terms of variability and skewness, the

Table 2.7
Range and Mean of Error Amounts in Five
Error Pools for Population 2

		Error Range		
Error Pool	Book Amount	Minimum*	Maximum*	Mean
1	0– 100.00	– 195.60	61.20	– 6.84
2	100.01– 500.00	– 459.60	354.00	–37.36
3	500.01– 2,000.00	–1,233.60	1,740.00	5.16
4	2,000.01– 5,000.00	–1,476.00	759.60	–14.26
5	5,000.01–10,000.00	–2,595.60	1,335.60	–99.85

*The extremes have been increased by 20% over the actual extreme amounts.

main difference (aside from the error rate) being that the 70% error rate population has a slightly higher mean than the other four study populations because it contains more errors.

Table 2.8
Major Characteristics of Audit Values in
Study Populations for Population 2

	Population Error Percentage				
Characteristic	.5	1	5	10	70
Total audit value	$3,487,012.00	$3,485,576.00	$3,490,751.00	$3,490,954.00	$3,564,610.00
Mean	$ 636.08	$ 635.82	$ 636.77	$ 636.80	$ 650.24
Standard deviation	$ 1,155.95	$ 1,155.84	$ 1,158.89	$ 1,158.88	$ 1,176.97
Skewness	3.5	3.5	3.5	3.5	3.5
Kurtosis	15.2	15.3	15.5	15.5	15.5

Population 3

Population 3 consists of the 7,026 trade accounts receivable of a second medium-size manufacturer. Accounts with balances over $100,000 were excluded because they will likely be audited on a 100% basis. There are only 27 of these accounts, yet they represent about one-third of the total book value.

Characteristics of Book Values. Table 2.9 (p. 22) presents a frequency distribution of the book amounts of the 7,026 trade accounts as well as the major characteristics of these book values. Figure 2.3 (p. 23) presents a frequency polygon of the distribution of book amounts. The distribution is highly skewed to the right, even somewhat more so than population 1M for which the largest accounts were also eliminated. Still, population 3 is not as skewed as population 1, even though the account balances for this manufacturer are much larger than those for the freight company.

Characteristics of Errors. An audit of 123 accounts with book balances under $100,000 disclosed nine errors, or an error rate of 7.3%. Clearly, such a small number of errors precludes obtaining any definitive information about the error pattern. The limited data available show the following pattern:

1. There is no major difference in the error rates for large and small accounts.

Table 2.9
Frequency Distribution and Major Characteristics
of Population 3 Book Values

Book Amount		Number of Accounts
0– 40.00		1,334
40.01– 136.00		1,438
136.01– 400.00		1,475
400.01– 800.00		878
800.01– 1,400.00		539
1,400.01– 3,000.00		548
3,000.01– 5,000.00		278
5,000.01– 10,000.00		239
10,000.01– 49,000.00		258
49,000.01–100,000.00		39
	Total	7,026
Total book value		$13,671,500.00
Mean		$ 1,945.84
Standard deviation		$ 7,021.61
Skewness		7.9
Kurtosis		78.1
Maximum		$ 98,162.70
Minimum		$ .10

2. The errors are all overstatements.

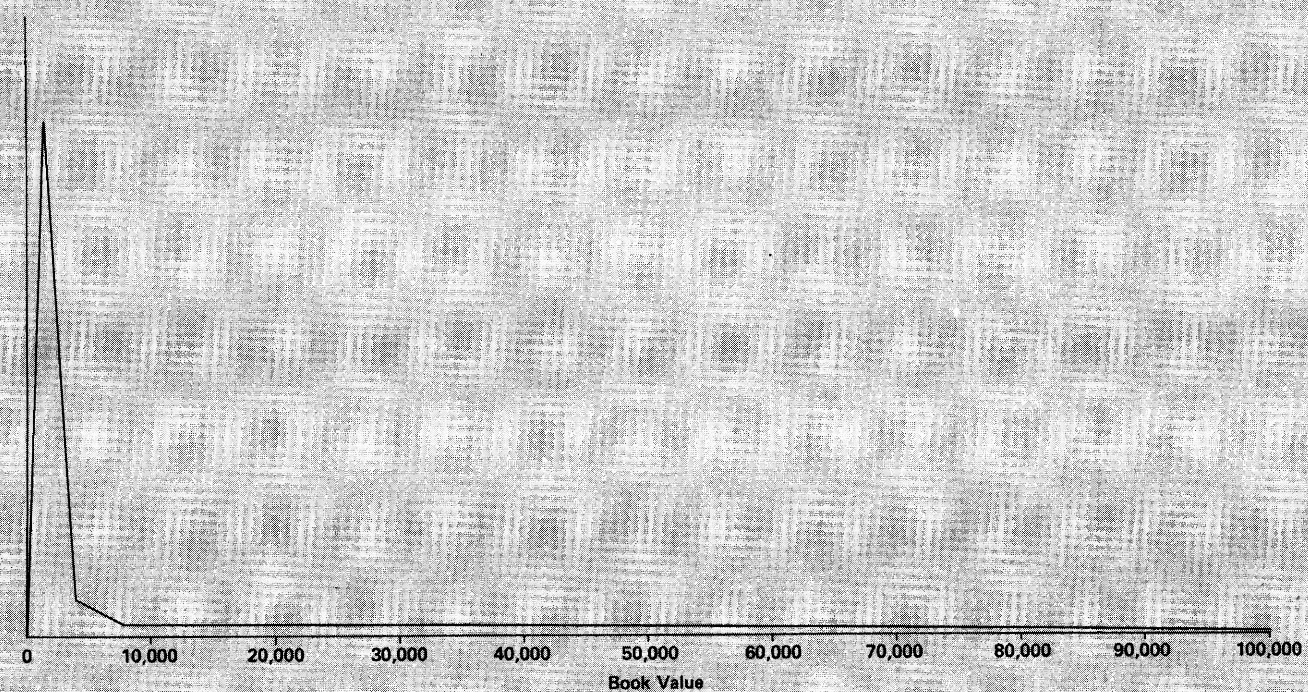
3. Small accounts have larger relative overstatements than large accounts. Three out of five accounts with balances under $500 have 100% overstatement errors, while none of the four accounts with balances of $500 or more have a relative error exceeding 50% (indeed, three out of these four relative errors are under 13%).

Creation of Study Populations. Five study populations, with error percentages of .5, 1, 5, 10, and 30, respectively, were created by utilizing the limited information on percent overstatements obtained from the audit. The procedure to create the 30% error rate study population was as follows:

1. From the 7,026 trade accounts in population 3, 2,107 trade accounts were selected at random.

2. The overstatement percentage multiple was then obtained from the appropriate error pool in Table 2.10 (p. 24) in accordance with the probabilities indicated there, and the error amount was calculated. If the

Figure 2.3 Frequency Polygon of Population 3 Book Values



error amount exceeded $500, it was modified to be $500 since none of the errors in the audit study exceeded $435.

The other study populations were obtained by randomly deleting errors from the 30% error rate study population.

Characteristics of Study Populations. The major characteristics of the audit values of the five study populations are shown in Table 2.11, opposite. All five study populations are quite similar in variability and skewness, with the mean declining slightly as the error rate increases.

Table 2.10
Overstatement Percentages in Error Pools
for Population 3

Percentage	Probability
Book Amount Under $200	
1	.06
2	.13
5	.06
50	.13
75	.13
100	.50
Book Amount Between $200 and $1,000	
5	.17
10	.08
15	.17
20	.08
50	.25
95	.17
100	.08
Book Amount Exceeding $1,000	
.01	.17
.05	.17
.1	.17
.2	.17
.3	.17
.5	.17

NOTES:
1. Overstatement percentage (in decimal form) times book value equals amount of overstatement error.
2. Probability values are rounded.

Table 2.11
Major Characteristics of Audit Values in
Study Populations for Population 3

Characteristic	Population Error Percentage .5	1	5	10	30
Total audit value	$13,668,964.00	$13,666,230.00	$13,647,829.00	$13,622,796.00	$13,509,839.00
Mean	$ 1,945.48	$ 1,945.09	$ 1,942.48	$ 1,938.91	$ 1,922.84
Standard deviation	$ 7,021.69	$ 7,021.77	$ 7,021.87	$ 7,022.59	$ 7,022.99
Skewness	7.9	7.9	7.9	7.9	7.9
Kurtosis	78.1	78.1	78.1	78.1	78.0

Population 4

Population 4 consists of 4,033 trade accounts receivable, which are a random subset of all trade accounts of a large manufacturer. Accounts with book balances over $25,000 are not included in population 4 because there were very few of these and they accounted for about 75% of the total book value. Hence, the auditor would probably examine them on a 100% basis.

Characteristics of Book Values. Table 2.12 (p. 26) contains a frequency distribution of the book values of the trade accounts receivable in population 4, and also the major characteristics of these book values. Figure 2.4 (p. 27) presents a frequency polygon of the distribution of book amounts. The distribution, as all others included in this study, is skewed to the right. The magnitude of skewness is nearly the same as for the inventory items in population 2 and is substantially less than that for the accounts in populations 1, 1M, and 3.

Characteristics of Errors. An audit of 174 trade accounts with book balances of $25,000 or less found ten balances in error, or an error rate of 5.7%. This is approximately the same error rate as for the trade accounts of the population 3 manufacturer. The few errors found in the study clearly preclude firm conclusions about the error pattern in the population. The ten errors show a pattern with the following characteristics:

1. There are no major differences in the error rates for large and small book amounts.
2. All errors are overstatements.
3. Seven of the ten errors are 100% overstatements; most of these are for smaller accounts.

4. Overstatement percentages, other than the 100% ones, are small—from 15% down to .5%.

Creation of Study Populations. Five study populations were created—with error percentages of .5, 1, 5, 10, and 30, respectively—on the basis of the limited information available about the error pattern. The procedure to create the 30% error rate study population was as follows:

1. From the 4,033 trade accounts in population 4, 1,209 trade accounts were selected at random.

Table 2.12
Frequency Distribution and Major Characteristics of Population 4 Book Values

Book Amount	Number of Accounts
0- 90.00	1,070
90.01- 230.00	715
230.01- 400.00	450
400.01- 650.00	337
650.01- 1,500.00	455
1,500.01- 3,500.00	409
3,500.01- 5,000.00	149
5,000.01-10,000.00	238
10,000.01-25,000.00	210
Total	4,033

Total book value	$7,502,957.00
Mean	$ 1,860.39
Standard deviation	$ 3,865.13
Skewness	3.2
Kurtosis	11.4
Maximum	$ 24,928.60
Minimum	$ .10

2. The overstatement percentage multiple was then obtained by choosing one from the appropriate error pool shown in Table 2.13 (p. 28) in accordance with the indicated probabilities, and the error amount was calculated.

The other study populations were obtained by randomly deleting errors from the 30% error rate study population.

Figure 2.4 Frequency Polygon of Population 4 Book Values

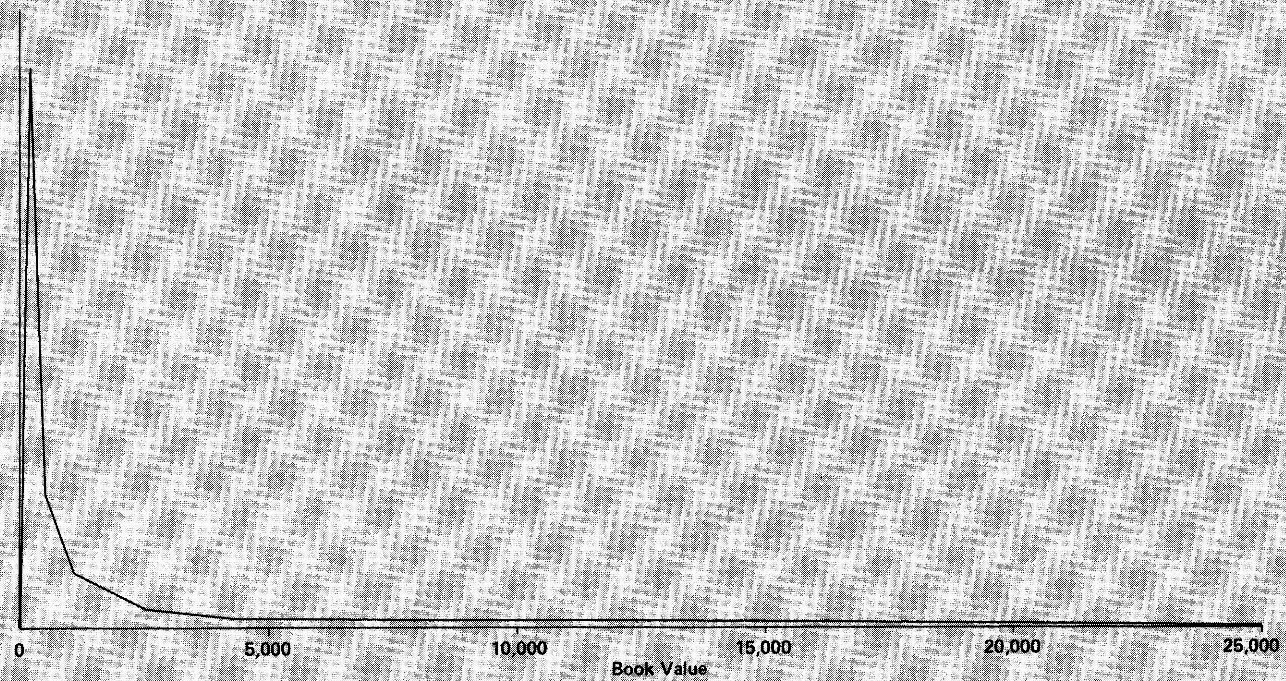


Table 2.13
Overstatement Percentages in Error Pools
for Population 4

	Probability	
Percentage	Book Amount Under $1,000	Book Amount $1,000 or More
.5	.02	.04
1.0	.02	.04
1.5	.02	.04
2.0	.02	.04
2.5	.02	.04
3.0	.02	.04
3.5	.02	.04
4.0	.02	.04
4.5	.02	.04
5.0	.02	.04
7.0	.02	.04
9.0	.02	.04
11.0	.02	.04
13.0	.02	.04
15.0	.02	.04
100.0	.70	.40

NOTE: Overstatement percentage (in decimal form) times book value equals amount of overstatement error.

Characteristics of Study Populations. Table 2.14, below, presents the major characteristics of the audit values of the five study populations. The five populations are very similar in terms of skewness, but the ones with high

Table 2.14
Major Characteristics of Audit Values in
Study Populations for Population 4

	Population Error Percentage				
Characteristic	.5	1	5	10	30
Total audit value	$7,478,146.00	$7,468,741.00	$7,402,350.00	$7,237,279.00	$6,442,371.00
Mean	$ 1,854.24	$ 1,851.91	$ 1,835.44	$ 1,794.52	$ 1,597.41
Standard deviation	$ 3,861.74	$ 3,861.20	$ 3,855.28	$ 3,813.98	$ 3,607.56
Skewness	3.3	3.3	3.3	3.3	3.5
Kurtosis	11.4	11.4	11.5	11.7	13.3

error rates have smaller means and standard deviations than those with small error rates. This is the result of the many 100% overstatement errors in the error pattern.

Results of Sampling Experiments in Appendix Tables

The basic results of the sampling experiments are presented in the Appendix in a series of tables, each of which consists of four sections.

Section 1. The first section of each table provides information on the sampling distribution of the estimated total audit value, denoted by $\hat{X}$.

1. *Mean.* This is the mean of the 600 estimates $\hat{X}$ obtained for the 600 samples in the experiment:

$$\bar{\hat{X}} = \frac{\sum_{i=1}^{600} \hat{X}_i}{600} \tag{2.2}$$

It is an estimate of the expected value of $\hat{X}$. A comparison of this mean with the true total audit value for the population (denoted by X) provides evidence of bias of the estimator and of the magnitude of this bias (if bias does exist).

2. *Standard deviation.* This is the standard deviation of the 600 estimates $\hat{X}$:

$$S(\hat{X}) = \left[\frac{\sum_{i=1}^{600} (\hat{X}_i - \bar{\hat{X}})^2}{600}\right]^{1/2} \tag{2.3}$$

It is an estimate of the standard error of $\hat{X}$ (conventionally denoted by $\sigma(\hat{X})$), which reflects the precision of the estimator $\hat{X}$.

3. *Skewness.* This measures the direction and degree of asymmetry in the 600 estimates $\hat{X}$:

$$Sk = \frac{\sum_{i=1}^{600} \left[\frac{\hat{X}_i - \bar{\hat{X}}}{S(\hat{X})}\right]^3}{600} \tag{2.4}$$

If the sampling distribution of $\hat{X}$ is normal, the measure Sk should be close to 0. If the sampling distribution of $\hat{X}$ is strongly skewed positively (the tail is to the right), the measure Sk should be a large positive number.

4. *Kurtosis*. This is another measure which is helpful in examining whether the sampling distribution of $\hat{X}$ is reasonably normal:

$$(2.5) \qquad Ku = \frac{\sum_{i=1}^{600} \left[\frac{\hat{X}_i - \overline{\hat{X}}}{S(\hat{X})} \right]^4}{600} - 3$$

If the sampling distribution of $\hat{X}$ is normal, the measure Ku should be close to 0. If the sampling distribution of $\hat{X}$ is markedly different from a normal distribution, Ku will be large either positively or negatively.

Section 2. In audit practice, confidence intervals for variables estimation are constructed by relying on approximate normality for reasonably large sample sizes. Thus, a two-sided confidence interval with a nominal confidence coefficient (reliability level) $1 - \alpha$ is constructed as follows:

$$(2.6) \qquad \hat{X} - z(1 - \alpha/2)s(\hat{X}) \leq X \leq \hat{X} + z(1 - \alpha/2)s(\hat{X})$$

where $\hat{X}$ is the point estimate of the total audit value for the population determined from the given sample, $z(1 - \alpha/2)$ is the $(1 - \alpha/2)100$ percentile of the standard normal distribution, and $s(\hat{X})$ is the estimated standard error of $\hat{X}$ determined from the given sample. Similarly, a one-sided lower confidence interval with a nominal confidence coefficient $1 - \alpha$ is constructed as follows:

$$(2.7) \qquad X \geq \hat{X} - z(1 - \alpha)s(\hat{X})$$

The actual confidence coefficient for these confidence intervals depends on the distribution of the standardized statistic:

$$(2.8) \qquad Z = \frac{\hat{X} - X}{s(\hat{X})}$$

If this distribution is exactly normal with mean 0 and standard deviation 1, the actual confidence coefficient is precisely the same as the nominal confidence coefficient $1 - \alpha$. The more the distribution of Z differs from that of a normal distribution with mean 0 and standard deviation 1, the greater will be the discrepancy between the actual confidence coefficient and the nominal one. Since auditors rely on the nominal confidence coefficient for an indication of the degree of assurance which they have that a confidence interval for a given sample will be correct, it is essential that the sampling distribution of Z be examined to see if it is approximately normal with mean 0 and standard deviation 1.

The second section of each Appendix table, therefore, provides the main characteristics of the sampling distribution of Z. The mean, standard deviation, measure of skewness, and measure of kurtosis are defined in corresponding fashion to the statistics for the distribution of $\hat{X}$. If the distribution of Z is approximately normal with mean 0 and standard deviation 1, the statistics Z_i from the 600 repetitions of the sampling experiment should have the following characteristics:

Mean: near 0
Standard deviation: near 1
Sk: near 0
Ku: near 0

For some estimators, the estimated standard error $s(\hat{X})$ equals zero when there are no errors in the sample. Since the standardized statistic Z is not defined in this case, the results in the second section of the Appendix tables are based only on those samples for which $s(\hat{X})$ does not equal zero.

Section 3. The third section of each Appendix table presents the coefficient of correlation between the estimate $\hat{X}$ and the estimated standard error $s(\hat{X})$ based on a given sample, in the 600 repetitions of the sampling experiment. It has been suggested that the behavior of this correlation coefficient may provide insights as to when the sampling distribution of Z is far from normal with mean 0 and standard deviation 1.[3]

Section 4. The Appendix tables do not contain chi-square or similar statistics measuring the departure of the actual sampling distribution of Z from the normal distribution with mean 0 and standard deviation 1 because serious departures are usually readily apparent from the descriptive statistics for the distribution of Z presented in the second section of the Appendix tables. Furthermore, interest centers not so much on true normality as on approximate normality which yields an actual confidence coefficient close to the nominal level based on the normal distribution. Hence, the fourth section of each Appendix table presents information for various types of confidence intervals on the proportion of the 600 confidence intervals in the sampling experiment which are correct. The following types of confidence intervals were studied:

Two-sided confidence interval—formula (2.6):
- 95.4% nominal confidence coefficient
- 98.8% nominal confidence coefficient

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3. Robert S. Kaplan, "Statistical Sampling in Auditing with Auxiliary Information Estimators," *Journal of Accounting Research,* Autumn 1973, pp. 238–258.

One-sided lower confidence interval—formula (2.7):
93.3% nominal confidence coefficient
97.7% nominal confidence coefficient

If the proportion of correct confidence intervals in the sampling experiment differs substantially from the corresponding nominal confidence coefficient, the implication would be that the auditor cannot rely on the nominal confidence coefficient for an indication of the actual degree of assurance which the estimation procedure provides.

Relative Standard Error

The analysis of the precision of the various estimators investigated utilizes the concept of the relative standard error. It is simply the estimated standard error of the estimator, $S(\hat{X})$, expressed as a percent of the true total audit value for the population, X. Thus:

$$(2.9) \qquad \text{Relative standard error} = \frac{S(\hat{X})}{X}100$$

An advantage of using the relative standard error over the standard error $S(\hat{X})$ is that it is frequently more meaningful to compare the results from different populations in terms of relative variability. In comparing the results for two populations, for example, the standard error $S(\hat{X})$ may differ simply because the observations in one case range from \$1 to \$1,000 and in the other case range from \$1,000 to \$1,000,000. The relative standard error, on the other hand, expresses the variability among the sample estimates $\hat{X}$ relative to their mean level.

Part II

Simple Random Sampling of Audit Units

3

Mean-Per-Unit Estimator

Simple Random Sampling of Audit Units

An audit unit is the basic unit of study by the auditor. In an examination of accounts receivable, the audit unit sometimes may be an individual account, at other times, an individual invoice. Similarly, in a study of inventory the audit unit may be an individual inventory item.

Simple random sampling of audit units (without replacement) is a selection procedure which gives every possible combination of n audit units which can be formed from the N audit units in the population an equal chance of being the sample combination chosen. A necessary, though not sufficient, requirement for a sample of audit units to be a simple random one is that each audit unit in the population has an equal probability of being included in the sample. A table of random numbers or computer generation of random numbers is usually employed to identify the audit units to be included in the simple random sample.

Mean-Per-Unit Estimator

Let $x_1, x_2, \ldots, x_n$ denote the audit values for the n audit units in the simple random sample. The mean-per-unit estimator of the population total audit value is then:

$$(3.1) \qquad \hat{X} = \frac{N}{n} \sum_{i=1}^{n} x_i = N\bar{x}$$

where $\bar{x}$ is the sample mean audit value:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3.2}$$

The estimated variance of the mean-per-unit estimator is:

$$s^2(\hat{X}) = N^2\left(1 - \frac{n}{N}\right)\frac{s^2}{n} \tag{3.3}$$

where s^2 is the sample variance:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1} \tag{3.4}$$

The estimated standard error of the estimator, denoted $s(\hat{X})$, is simply the positive square root of $s^2(\hat{X})$.

Experimental Findings

The basic experimental results for the mean-per-unit estimator with simple random sampling of audit units are presented in Appendix Tables A-1 through A-10 (pp. 143–147). The principal reason for including this estimator in the experimental study was to examine, for sample sizes often encountered in auditing, the reliability of the nominal confidence coefficient based on normality—that is, how close the nominal confidence coefficient is to the actual confidence coefficient.

It was unnecessary to study the bias of the mean-per-unit estimator because statistical theory indicates that this estimator is unbiased. Indeed, the experimental results for each study population show that the mean of the 600 $\hat{X}$ values is close (and well within sampling error margins at a confidence level of 95%) to the true total audit value.

Precision of Mean-Per-Unit Estimator. Practice has shown that the mean-per-unit estimator is highly inefficient for accounting populations. Table 3.1, opposite, shows the standard error of $\hat{X}$ relative to the true total audit value for sample sizes 100 and 200 for each study population, and the results are in accord with this experience. The large values of the relative standard error indicate that the sampling distributions of the estimator $\hat{X}$ are highly variable relative to the true total audit value.

Table 3.1
Relative Standard Error of $\hat{X}$
(in percent)

Population	.5	1	5	10	30*
			n = 100		
1	24.1	24.0	–	24.0	23.9
1M	–	17.9	17.9	17.9	–
2	18.2	18.2	18.3	18.2	18.3
3	35.7	35.7	35.7	35.8	36.1
4	20.2	20.2	20.4	20.7	21.4
			n = 200		
1	19.2	19.2	–	19.1	19.1
1M	–	12.2	12.2	12.2	–
2	12.4	12.4	12.5	12.4	12.2
3	26.4	26.4	26.4	26.5	26.7
4	14.1	14.1	14.3	14.5	15.2

*For population 2, this error percentage is 70.

Reliability of Nominal Confidence Coefficient. Tables 3.2 and 3.3 (pp. 38 and 39) show the percent of correct interval estimates for two-sided, nominal 95.4% confidence intervals and for one-sided lower, nominal 93.3% confidence intervals. Table 3.2, for sample size 100, shows clearly that the actual proportions of the two-sided confidence intervals which are correct (that is, which include the population total audit value) are smaller than the nominal 95.4% confidence coefficient for all populations studied.[1] The actual proportions of correct intervals are substantially below the nominal level for populations 1, 1M, and 3 (the most highly skewed populations), but are only slightly below the nominal level for the other two

1. The results for the different error rate study populations for a given population and a given sample size are not independent because the same seed was used in generating the random numbers in each case. On the other hand, the results for sample sizes 100 and 200 are independent because different seeds were used. The lack of independence between the results for the different error rate study populations has no effect on the evaluation for any one study population. Only when the results for different error rate study populations for a given population are compared must this lack of independence be considered.

Table 3.2
Actual Percent of Correct Confidence Intervals
With Mean-Per-Unit Estimator, $n = 100$

Population	Population Error Percentage .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	81.8	81.8	–	81.7	81.7
1M	–	89.8	89.8	89.2	–
2	93.7	93.7	93.7	93.7	93.0
3	82.5	82.5	82.5	82.5	82.5
4	92.7	92.7	92.3	92.8	93.2
	One-sided Lower, Nominal 93.3%				
1	99.2	99.2	–	99.2	99.2
1M	–	98.0	98.0	98.0	–
2	96.5	96.3	96.7	96.5	96.7
3	98.0	98.0	98.0	98.0	98.0
4	96.5	96.5	96.5	96.8	96.8

*For population 2, this error percentage is 70.

populations (the more moderately skewed ones).

Conversely, the proportions of correct intervals for the one-sided lower confidence limit exceed the nominal 93.3 percent confidence coefficient for all populations. Again, the differences between the actual and nominal levels are greater for the highly skewed populations and are smaller for the more moderately skewed populations.

The results for sample size 200 in Table 3.3 correspond to those for sample size 100, the major difference being that the discrepancies between the actual proportion correct and the nominal confidence coefficient are smaller. Indeed, for the more moderately skewed populations 2 and 4 there are practically no discrepancies for the two-sided confidence interval and only small discrepancies for the one-sided interval.

The overstatement by the nominal confidence coefficient of the actual assurance level for the two-sided interval and the understatement for the one-sided lower confidence interval are associated with the positive skewness of the population. This leads, for moderate sample sizes, to a sampling distribution of $\widehat{X}$ which is also skewed positively. Further, the positive population skewness leads to a positive correlation between $\widehat{X}$ and the estimated standard error of $\widehat{X}$ as shown in the Appendix tables. This latter result occurs since extreme observations from the right tail of the distribution tend to increase the sample mean as well as the sample standard deviation. In turn, the positive skewness of the distribution of $\widehat{X}$, together with the positive correlation between $\widehat{X}$ and the estimated standard error of $\widehat{X}$, lead to a negatively skewed distribution of Z.[2] As the sample size becomes large, the skewness of the distribution of Z disappears and the actual proportion of correct intervals approaches the nominal confidence coefficient based on the normal distribution.

We have not discussed explicitly the results for the other two confidence intervals presented in the Appendix tables (two-sided, nominal 98.8%, and one-sided lower, nominal 97.7%) because they show the same patterns as the two confidence intervals just discussed.

Table 3.3
Actual Percent of Correct Confidence Intervals
With Mean-Per-Unit Estimator, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	87.3	87.5	–	87.2	87.2
1M	–	91.8	92.3	92.3	–
2	95.3	95.5	95.7	95.7	95.5
3	87.5	87.7	87.7	87.7	87.7
4	95.3	95.2	95.2	95.0	94.7
	One-sided Lower, Nominal 93.3%				
1	98.2	98.3	–	98.3	98.2
1M	–	97.8	97.8	97.7	–
2	94.8	94.5	95.0	94.3	95.3
3	97.2	97.2	97.2	97.2	97.2
4	95.3	95.0	95.0	95.0	95.7

*For population 2, this error percentage is 70.

2. As noted earlier, statistical tests can be used for examining the normality of the sampling distribution of Z. For example, the chi-square test applied to the distribution of Z for the .5% error rate study population for population 1 rejects, at level of significance .01, the hypothesis that the distribution is normal with mean 0 and standard deviation 1. This result is not surprising because the estimated mean of this distribution is $-.7$, with estimated standard error $1.7/\sqrt{600} = .07$, which suggests that the mean of the distribution is not zero.

4

Difference and Ratio Estimators

When an auditor utilizes statistical sampling, he generally has available not only information on the audit values of the audit units in the sample, but also has information on their book values and on the total book value of the population. Use of this auxiliary information by means of difference and ratio estimators frequently leads to much more precise estimates than with the mean-per-unit estimator which utilizes only information on the audit values of the sample units. A regression estimator also utilizes auxiliary information. It is an extension of the ratio estimator, but it was not included in this study because it is not used as frequently as the ratio estimator.

Difference Estimator

Let $x_1, x_2, \ldots, x_n$ denote the audit values for the n audit units in the simple random sample of audit units, and $y_1, y_2, \ldots, y_n$ their corresponding book values. Further, let Y denote the population total book value:

$$(4.1) \qquad Y = \sum_{i=1}^{N} Y_i$$

The population total audit value is denoted as before by X. The difference estimator of this total audit value is:

$$(4.2) \qquad \hat{X} = Y + N(\bar{x} - \bar{y})$$

where:

$$(4.3a) \qquad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$(4.3b) \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

When no errors are found in the sample, $\bar{x}$ equals $\bar{y}$ and the difference estimate of the population total audit value is simply Y, the population total book value.

The estimated variance of this difference estimator is:

$$(4.4) \qquad s^2(\hat{X}) = N^2\left(1 - \frac{n}{N}\right)\frac{1}{n(n-1)}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{i=1}^{n}(y_i - \bar{y})^2 - 2\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right]$$

When there are no errors in the sample audit units, $s^2(\hat{X})$ equals zero. The estimated standard error of the estimator, denoted as before $s(\hat{X})$, is the positive square root of the estimated variance in formula (4.4).

Ratio Estimator

The ratio estimator of the population total audit value is:

$$(4.5) \qquad \hat{X} = \frac{\bar{x}}{\bar{y}} Y$$

where $\bar{x}$ and $\bar{y}$ are the sample mean audit and book values, respectively, as defined in formula (4.3). Like the difference estimate, the ratio estimate of the population total audit value when no errors are found in the sample is Y, the population total book value.

The ratio estimator is biased, that is, in repeated sampling the mean of the estimates does not equal the population total audit value. However, the bias becomes small for large sample sizes.

The estimated variance of the ratio estimator is:

$$(4.6) \qquad s^2(\hat{X}) = N^2\left(1 - \frac{n}{N}\right)\frac{1}{n(n-1)}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + r^2\sum_{i=1}^{n}(y_i - \bar{y})^2 - 2r\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right]$$

where r is the sample ratio:

$$(4.7) \qquad r = \frac{\bar{x}}{\bar{y}}$$

This estimated variance is biased, but the bias is small for large sample sizes. The positive square root of the estimated variance is the estimated standard error of the ratio estimator.[1]

As with the difference estimator, the estimated standard error of the ratio estimator equals zero when no errors are found in the random sample.

Experimental Findings

The basic experimental results are presented in Appendix Tables A-11 through A-30 (pp. 148–167).

Bias of Ratio Estimator. For the populations and sample sizes considered in this empirical study, the ratio estimator has negligible bias. Table 4.1 (p. 44) presents, for each study population, the mean value of the 600 ratio estimates for sample size 100 as a percent of the population total audit value. In no case does this percentage depart markedly from 100.0. While a few of the differences between the mean of the 600 ratio estimates and the population total audit value are statistically significant (at level .05), indicating that bias is indeed present, the results in Table 4.1 suggest that any such bias is negligibly small. The results for sample size 200 are not presented because the bias of the ratio estimator must be even smaller according to statistical theory for that sample size than it is for sample size 100.

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1. Even if the estimated variance were not biased, the estimated standard error would contain some bias.

Table 4.1
Mean Value of 600 Ratio Estimates as Percent of Population Total Audit Value, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
1	100.0	100.0	100.0	100.0	100.1
1M	–	100.0	100.0	100.0	–
2	100.0	100.0	100.1	100.0	100.2
3	100.0	100.0	100.0	100.0	99.8
4	100.1	100.0	100.0	100.0	100.2

*For population 2, this error percentage is 70.

Precision of Difference and Ratio Estimators. Table 4.2, below, presents, for each study population, the standard error of the difference estimator as a percentage of the true total audit value for sample size 100, and Table 4.3, opposite, does likewise for the ratio estimator. A number of interesting results are apparent from these tables as follows:

1. The precisions of the difference and ratio estimators are practically the same for the populations considered. There are some variations, to be sure. For instance, the difference estimator appears to be somewhat more precise than the ratio estimator for the higher error rate study populations for population 3. On the other hand, the ratio estimator seems to be somewhat more precise than the difference estimator for the 30% error rate study population for population 4. But these differ-

Table 4.2
Relative Standard Error of Difference Estimator, $n = 100$ (in percent)

	Population Error Percentage				
Population	.5	1	5	10	30*
1	.1	.2	.4	.6	.9
1M	–	.2	.5	.6	–
2	.2	.4	1.0	1.3	3.0
3	.1	.1	.1	.2	.4
4	1.1	1.1	1.6	3.5	9.2

*For population 2, this error percentage is 70.

ences in precision are small compared to those when the ratio and difference estimators are contrasted with the mean-per-unit estimator.

It should be cautioned that the similarity in precisions of the ratio and difference estimators for the four populations studied need not hold for some other populations with different characteristics and error patterns.

2. A comparison of Tables 4.2 and 4.3 with the results in Table 3.1 (p. 37) for sample size 100 shows clearly that the ratio and difference estimators are much more precise than the mean-per-unit estimator, for the populations considered.

3. While the relative standard error for the mean-per-unit estimator tends not to change much as the population contains higher error rates (Table 3.1), this is not the case for the difference and ratio estimators. For these estimators, the relative standard error increases as the error rate increases. Thus, difference and ratio estimators yield relatively less precise estimates when the error rate is high than when it is low, for the populations investigated.

Table 4.3
Relative Standard Error of Ratio Estimator, $n = 100$
(in percent)

	Population Error Percentage				
Population	.5	1	5	10	30*
1	.1	.2	.4	.6	.9
1M	–	.2	.5	.6	–
2	.2	.4	1.0	1.3	3.1
3	.1	.1	.2	.3	.7
4	1.1	1.1	1.6	3.5	8.4

* For population 2, this error percentage is 70.

It should be noted here that the increase in the error rate is not the direct causal factor leading to an increase in the relative standard errors. For the difference estimator, for example, the increase in the error rate leads to greater variability among the errors (including zeros for no errors) in the populations studied, but this need not always be the case. Further, the development of the study populations assumed the same error pattern for all error rates, and this need not always be the case. It will be convenient, however, to continue to refer to changes "as the error rate increases" with the understanding that the error rate simply is used as an index for a study population.

4. The difference and ratio estimators are least precise, relatively, for populations 2 and 4. These are the two populations with the largest error amounts. The errors for the population of inventory items (population 2), it will be recalled, are relatively large. However, these errors are not as large as those for the trade accounts of the population 4 manufacturer which frequently are 100% overstatements even for large accounts. As might be expected from the difference in the magnitudes of the errors,[2] the relative precisions of the difference and ratio estimators for population 4 are substantially worse than those for population 2.

The results on relative precision for sample size 200 are not discussed explicitly because they follow those for sample size 100.

Percentage of Samples With Errors. As noted earlier, the estimated standard errors for both the difference and ratio estimators equal zero when no errors are found in the sample. This is a meaningless result for the auditor since he cannot interpret it as implying that the estimate is a perfect one.

Appendix Tables A-11 through A-30 show the number of samples out of the 600 repetitions for each study population which contains at least one error, thus leading to an estimated standard error which is not zero.[3] Table 4.4, opposite, shows the extent of the problem of samples containing no errors, showing the percentage of the 600 samples containing one or more errors for each study population. For sample size 100, more than half of the samples contain no errors when the population error rate is .5%, and about one-third of the samples contain no errors when the error rate is 1%. For larger error rates, from 5% on, almost all samples contain at least one error. The results for sample size 200 are similar, except that a smaller proportion of samples contains no errors for the lower error rates.

These results—that for sample sizes of 100 and 200, a substantial proportion of samples leads to an estimated standard error of zero for the difference and ratio estimators when the population error rate is small—are not unexpected. Indeed, probability theory can furnish the exact probability that a sample of given size will contain no errors for a specified population error rate. For example, when the error rate is .5%, the exact probability that no errors are present in a random sample of 100 audit units from population

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2. The larger magnitudes of errors for population 4 are associated with a somewhat lower correlation between the book and audit values than is the case for population 2 where the error amounts are smaller.
3. Since the ratio and difference estimators are based on the same samples, the number of samples with one or more errors for any given study population is the same for both estimators.

Table 4.4
Percent of 600 Simple Random Samples Containing
One or More Errors

Population	Population Error Percentage .5	1	5	10	30*
			$n = 100$		
1	38.0	61.5	99.0	100.0	100.0
1M	–	61.3	99.5	100.0	–
2	41.3	67.0	99.7	100.0	100.0
3	38.7	62.8	99.5	100.0	100.0
4	41.8	64.3	99.3	100.0	100.0
			$n = 200$		
1	64.3	86.5	100.0	100.0	100.0
1M	–	86.5	100.0	100.0	–
2	62.7	88.3	100.0	100.0	100.0
3	62.8	85.5	100.0	100.0	100.0
4	64.0	87.8	100.0	100.0	100.0

*For population 2, this error percentage is 70.

1 is 60.6%. The actual proportion of the 600 samples in the experiment which contain no errors is 62.0%.[4] The empirical results are presented to show the dimensions of the problem in a readily understandable context.

Reliability of the Nominal Confidence Coefficient. Difference and ratio estimators are not only subject to the problem of an estimated standard error of zero when the sample contains no errors. There also may be a problem in using the usual confidence intervals if the distribution of the standardized statistic Z is not approximately normal with mean 0 and standard deviation 1. This latter problem is particularly to be expected when samples tend to contain only a few errors and the estimated standard error then varies erratically from sample to sample.

Information on the distribution of Z for the difference and ratio estimators is contained in Appendix Tables A-11 through A-30 (pp. 148–167). Since

4. Indeed, comparisons between the actual proportions and the theoretical probabilities were utilized as a check on the computer random selection procedure.

the statistic Z is not defined when the estimated standard error $s(\hat{X})$ equals zero, the characteristics of the distribution of Z shown in the Appendix tables are based on only those samples containing one or more errors.

Tables 4.5 through 4.8 (pp. 48–51) show the combined effects of samples leading to an estimated standard error of zero and of failure of the distribution of Z to be a standard normal distribution. These tables show the percentage of the confidence intervals that are "correct" in the 600 sample repetitions for a study population. Results for two types of confidence intervals are presented in these tables—namely, for a two-sided, nominal 95.4% confidence interval and for a one-sided lower, nominal 93.3% confidence interval. A confidence interval is considered to be "correct" if the estimated standard error is not zero and if the limit(s) include(s) the population total audit value.

A number of interesting findings are prominent in Tables 4.5 through 4.8 as follows:

1. The proportions of correct confidence intervals are quite similar for the difference and ratio estimators. To be sure, there appear to be some differences in behavior between these two estimators, but these are not large enough to change the overall picture of similarity of behavior.

Table 4.5
Actual Percent of Correct† Confidence Intervals
With Difference Estimator, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	30.5	37.3	96.8	94.0	96.3
1M	–	38.8	97.5	93.8	–
2	31.2	41.8	82.3	97.2	95.5
3	23.3	36.8	73.7	80.3	90.8
4	21.2	30.0	58.2	62.0	74.8
	One-sided Lower, Nominal 93.3%				
1	38.0	61.3	92.8	97.3	94.3
1M	–	61.3	92.3	98.8	–
2	41.3	34.5	98.7	96.8	94.8
3	23.3	36.2	68.5	75.8	87.7
4	17.3	29.3	53.8	56.5	69.3

†If estimated standard error is zero, interval is considered incorrect.
*For population 2, this error percentage is 70.

Table 4.6
Actual Percent of Correct† Confidence Intervals
With Difference Estimator, $n = 200$

Population	Population Error Percentage .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	40.0	53.0	99.2	93.5	93.7
1M	–	53.2	99.2	93.0	–
2	44.0	54.5	90.2	97.0	96.0
3	42.3	57.0	82.3	87.7	92.2
4	28.0	47.0	69.7	70.7	87.5
	One-sided Lower, Nominal 93.3%				
1	64.3	86.3	96.5	98.2	95.0
1M	–	86.2	96.5	98.3	–
2	62.7	52.5	98.0	94.7	94.7
3	42.3	53.2	77.0	82.5	88.8
4	26.2	42.0	66.3	65.8	82.2

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

2. For the study populations with error rates of .5% and 1%, the proportions of correct intervals are so far below the nominal confidence coefficients, for both sample sizes 100 and 200, that the nominal confidence coefficients become meaningless. Only for populations 1 and 1M, where the errors are small and tend to balance out, are the proportions of correct one-sided lower intervals for sample size 200 anywhere near the nominal confidence coefficient of 93.3% for the 1% error rate study population.

   A low proportion of correct intervals does not, of course, necessarily imply that the estimates are imprecise. Many samples contain no errors and thus provide no confidence limits. They are, therefore, considered incorrect because the point estimate differs from the true population total, even though the estimate (which is the total book value in this case) is close to the true total audit value. In addition, some of the samples with errors yield such a small estimated standard error that the confidence interval does not include the true total audit value, even though the estimate is close to the total audit value. The auditor, of course, does not know the precision of the estimator from selecting a number of samples, as is possible in this experimental study. Instead, he must rely on the confidence interval based on the selected sample. Thus, the failure of confidence intervals to include the true total audit value is important to him even though the estimates may indeed be close to the true total audit value.

3. For populations 1, 1M, and 2, which have error patterns containing both overstatement and understatement errors, the proportions of correct two-sided intervals are close to the nominal 95.4% confidence coefficient for the 10%-and-higher error rate study populations with both sample sizes 100 and 200. Indeed, for populations 1 and 1M, where the error amounts are much smaller than those for population 2, this is already the case for the 5% error rate study populations. In these instances the proportion of correct intervals does not differ from the nominal confidence coefficient by more than 4.1 percent points.

4. With these same populations (1, 1M, and 2), the proportions of correct one-sided lower confidence limits are within 4.0 percent points below

Table 4.7
Actual Percent of Correct† Confidence Intervals
With Ratio Estimator, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	29.8	36.2	94.3	91.5	92.8
1M	–	39.8	96.2	93.7	–
2	30.7	41.7	82.2	96.5	94.2
3	24.2	35.2	72.8	79.5	86.2
4	21.0	31.0	59.3	63.3	76.0
	One-sided Lower, Nominal 93.3%				
1	37.7	60.8	89.3	94.0	90.8
1M	–	61.3	91.2	96.3	–
2	41.2	35.5	97.7	93.8	92.0
3	24.2	36.5	71.3	80.2	91.5
4	17.7	29.8	55.5	58.7	72.5

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 4.8
Actual Percent of Correct† Confidence Intervals
With Ratio Estimator, $n = 200$

Population	.5	1	5	10	30*
			Two-sided, Nominal 95.4%		
1	41.2	52.0	97.2	92.7	91.3
1M	–	53.0	99.0	93.2	–
2	44.0	55.0	89.2	96.5	94.3
3	41.8	57.0	81.8	86.3	91.3
4	30.3	47.3	70.2	71.2	87.5
			One-sided Lower, Nominal 93.3%		
1	64.0	85.7	95.0	94.7	92.0
1M	–	85.5	95.8	96.7	–
2	62.5	51.5	97.5	94.0	92.7
3	42.2	55.0	79.3	84.2	92.8
4	25.7	42.5	66.2	66.3	83.8

(Column heading: Population Error Percentage)

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

the nominal 93.3% confidence coefficient or above it for all study populations with error rates of 5% or more, for both sample sizes 100 and 200.

5. There is a tendency with these same populations (1, 1M, and 2) for the proportion of correct two-sided intervals to be slightly higher for the difference estimator than for the ratio estimator for study populations with error rates of 5% or more, for both sample sizes 100 and 200. Similarly, the proportion of correct one-sided lower intervals tends to be slightly higher for the difference estimator than for the ratio estimator in these cases.

6. For populations 3 and 4, which have error patterns containing only overstatements, the proportions of correct intervals are substantially below the nominal confidence coefficients for both the 5% and 10% error rate study populations, for both ratio and difference estimators, and both sample sizes 100 and 200. Even when the error rate is 30% in population 4, the proportions of correct two-sided intervals are still far below the nominal confidence coefficient of 95.4 percent for sample

size 100 and are almost 8 percent points below the nominal level for sample size 200. Only for the 30% error rate study population for population 3 (which contains smaller overstatement errors than population 4) does the proportion of correct two-sided intervals for the difference estimator come within 4.6 percent points of the nominal confidence coefficient for sample size 100 and within 3.2 percent points for sample size 200.[5]

Similarly, the proportions of correct one-sided lower confidence intervals are far below the nominal 93.3% level for the 5% and 10% error rate study populations for populations 3 and 4. Only for the 30% error rate study population for population 3 is the proportion of correct one-sided lower intervals fairly close to the nominal level for the difference and ratio estimators for both sample sizes.

7. Table 4.9, opposite, summarizes the problem of unreliability of the nominal confidence coefficients for the difference and ratio estimators. This table shows the first error rate for which the actual proportion of correct intervals is "close" to the nominal confidence level. The actual proportion is considered to be "close" to the nominal confidence level if it is within 5 percent points below or anywhere above the nominal level. Table 4.9 again shows that the nominal confidence coefficients are unreliable according to the criterion of "closeness" used here for all error rate study populations for population 4, and for all but the highest error rate study population for population 3.

The results for the two-sided, nominal 98.8% confidence interval and for the one-sided lower, nominal 97.7% confidence interval are not discussed explicitly here because they are similar to those for the two confidence intervals considered above.

It is interesting to note from Appendix Tables A-11 through A-30 that the correlations between the estimates $\hat{X}$ and the estimated standard errors $s(\hat{X})$ differ substantially from those for the mean-per-unit estimator. For the latter, the correlation coefficients are consistently high and positive. In contrast, for the difference and ratio estimators the correlations for populations 1, 1M, and 2 (where the errors go in both directions) are not too far from zero for

5. The poorer reliability of the nominal confidence coefficient for populations 3 and 4 is not associated directly with the correlation between the book and audit values. For example, these correlations for the highest error rate study populations for populations 1, 2, 3, and 4 are respectively 1.000, .986, 1.000, and .924. Thus, populations 3 and 4 are not distinguished by either comparatively low or high correlations.

the study populations with error rates of 5% or more. Differing even more, the correlations for populations 3 and 4 (where the errors are all overstatements) are large and negative.

Table 4.9
Error Rate for Which Proportion of Correct Intervals First is Close† to Nominal Level for Difference and Ratio Estimators

Population	$n = 100$	$n = 200$
	Two-sided, Nominal 95.4%	
1	5%	5%
1M	5%	5%
2	10%	10%
3	D-30%; R-None*	30%
4	None	None
	One-sided Lower, Nominal 93.3%	
1	5%	5%
1M	5%	5%
2	5%	5%
3	D-None; R-30%*	30%
4	None	None

† Close is defined to be within 5 percent points below nominal level or anywhere above it.
* D: difference estimator
R: ratio estimator

5

Combined Mean-Per-Unit and Auxiliary Information Estimators

It was noted in chapter four that the estimated standard errors of both the difference and ratio estimators equal zero when no errors are found in the sample. Unfortunately this is a frequent occurrence when the population error rate is low. To benefit from the high precision that difference and ratio estimators tend to have in this situation and yet avoid the problem of an estimated standard error of zero, Howard L. Jones has suggested the possibility of using a weighted average of the mean-per-unit estimator and either the difference or the ratio estimator.[1]

Combined Mean-Per-Unit and Difference Estimator

The combined mean-per-unit and difference estimator (for brevity to be called the mpu-difference estimator) is:

(5.1) $$\hat{X} = w(N\bar{x}) + (1 - w)[Y + N(\bar{x} - \bar{y})]$$

1. Howard L. Jones, "Developing Statistical Standards for External Audits," unpublished paper, 1972.

where w is a weight between 0 and 1. If $w = 0$, the estimator in formula (5.1) becomes simply the difference estimator, and if $w = 1$, it becomes the mean-per-unit estimator.

The estimated variance of $\hat{X}$ is:[2]

$$(5.2) \qquad s^2(\hat{X}) = N^2\left(1 - \frac{n}{N}\right)\frac{1}{n(n-1)}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + (1-w)^2\sum_{i=1}^{n}(y_i - \bar{y})^2 - 2(1-w)\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right]$$

When there are no errors in the sample, this estimated variance will generally not equal zero as long as the weight w is not zero.

Combined Mean-Per-Unit and Ratio Estimator

The combined mean-per-unit and ratio estimator (for brevity to be called the mpu-ratio estimator) is:

$$(5.3) \qquad \hat{X} = w(N\bar{x}) + (1 - w)\left(\frac{\bar{x}}{\bar{y}}Y\right)$$

If the weight $w = 0$, $\hat{X}$ is simply the ratio estimator and if $w = 1$, $\hat{X}$ becomes the mean-per-unit estimator. The estimator is biased when $w \neq 1$, but the bias becomes small for large sample sizes.

The estimated variance of the mpu-ratio estimator is:

$$(5.4) \qquad s^2(\hat{X}) = N^2\left(1 - \frac{n}{N}\right)\frac{1}{n(n-1)}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + (1-w)^2r^2\sum_{i=1}^{n}(y_i - \bar{y})^2 - 2(1-w)r\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right]$$

where:

$$(5.5) \qquad r = \frac{\bar{x}}{\bar{y}}$$

2. The estimated variance is not simply a combination of the estimated variances of the mean-per-unit and difference estimators, because these two estimators are based on the same sample data and are correlated. Formula (5.2) takes this correlation into account.

Again, this estimated variance generally does not equal zero when there are no errors in the sample, as long as the weight w is not zero. The estimated variance in formula (5.4) is biased when $w \neq 1$, but the bias becomes small when the sample size is large.

Choice of Weight

When the ratio or difference estimators are substantially more precise than the mean-per-unit estimator, a small weight w will utilize most of their efficiency. On the other hand, a small weight w may also lead to erratic behavior in the estimated standard error of the combined estimator when the population error rate is small, arising from the instability of the estimated standard error of the auxiliary information estimator.

To investigate the effect of the choice of weight on the precision and reliability of interval estimates, weights $w = .1$ and $w = .4$ were investigated for population 1. Appendix Tables A-31, A-32, A-41 through A-44, A-53 and A-54 (pp. 168–179) contain the results of this study. Tables 5.1, below, and 5.2 (p. 58) summarize the results for sample size 100. Table 5.1 presents data on the relative standard error. It readily shows that raising the weight from .1 to .4 increases the relative standard error about four-fold. Table 5.2 contains data on the percentage of correct intervals. No major differences are discernible in the results for weights .1 and .4. The results for sample size 200 follow the same pattern as those for sample size 100; they are not discussed in detail here.

Since an increase in weight from .1 to .4 does not substantially improve the reliability of the nominal confidence coefficient for population 1 and

Table 5.1
Relative Standard Errors of Combined Estimators for Population 1, $n = 100$
(in percent)

	Population Error Percentage				
Weight	.5	1	5	10	30
	MPU-Difference Estimator				
.1	2.4	2.4	–	2.5	2.5
.4	9.6	9.6	–	9.6	9.6
	MPU-Ratio Estimator				
.1	2.4	2.4	–	2.4	2.5
.4	9.6	9.6	–	9.6	9.5

Table 5.2
Actual Percent of Correct Confidence Intervals With Combined Estimators for Population 1, $n = 100$

Estimator and Weight	.5	1	5	10	30
	Population Error Percentage				
	Two-sided, Nominal 95.4%				
MPU-Difference					
.1	82.3	82.0	-	83.3	84.3
.4	81.8	82.0	-	81.8	82.2
MPU-Ratio					
.1	82.3	82.0	-	83.8	85.0
.4	81.8	82.0	-	81.8	82.2
	One-sided Lower, Nominal 93.3%				
MPU-Difference					
.1	99.2	99.2	-	99.2	99.3
.4	99.2	99.2	-	99.0	99.5
MPU-Ratio					
.1	99.2	99.2	-	99.2	99.3
.4	99.2	99.2	-	99.0	99.3

worsens the relative precision to a major extent, it was decided to conduct the full study with weight $w = .1$.

Experimental Findings

The basic experimental results are presented in Appendix Tables A-31 through A-40 and A-43 through A-52. (See pages 168–178.)

Bias of MPU-Ratio Estimator. It was noted in chapter four that the bias of the ratio estimator, for the populations and sample sizes considered here, is negligibly small. Since the mpu-ratio estimator involves only a fraction of the bias for the ratio estimator alone (90% of it with $w = .1$), there should be no noticeable bias effects in the experimental results. The data in the Appendix tables confirm this. The difference between the mean of the 600 sample estimates and the true total audit value never exceeds sampling variation (95% confidence level).

Precision of Combined Estimators. Table 5.3, below, presents, for each study population, the standard error of the mpu-difference estimator as a percentage of the true total audit value for sample size 100. Table 5.4 (p. 60) does likewise for the mpu-ratio estimator. The following interesting findings emerge from these tables:

1. The mpu-difference and mpu-ratio estimators have practically the same precisions for the populations under consideration. This is expected because the precisions of the difference and ratio estimators are about the same for the populations under study, as noted in chapter four.
2. The relative precisions of the combined estimators are far better than those of the mean-per-unit estimator for the same sample size (Table 3.1, p. 37).
3. The relative standard errors of the combined estimators tend to be larger than those of the difference and ratio estimators (Tables 4.2 and 4.3, pp. 44 and 45). However, for populations 2 and 4 (which have the largest error amounts), there is little difference between the relative standard errors for the largest error rate study population.
4. The increase in the relative standard error with higher error rates, noted earlier for the difference and ratio estimators, is much more moderate for the combined estimators.
5. The relative standard errors of the combined estimators are largest for population 3 when the error rate is 5% or less, and largest for population 4 when the error rate is 10% or more. This differs from the behavior of the difference and ratio estimators, where the relative standard errors for population 4 were much larger than those for population 3 for all error rates.

Table 5.3
Relative Standard Error of MPU-Difference Estimator, $n = 100$
(in percent)

	Population Error Percentage				
Population	.5	1	5	10	30*
1	2.4	2.4	–	2.5	2.5
1M	–	1.8	1.8	1.9	–
2	1.8	1.9	2.1	2.2	3.6
3	3.6	3.6	3.6	3.6	3.6
4	2.2	2.3	2.5	3.8	8.5

*For population 2, this error percentage is 70.

Table 5.4
Relative Standard Error of MPU-Ratio
Estimator, $n = 100$
(in percent)

Population	Population Error Percentage .5	1	5	10	30*
1	2.4	2.4	–	2.4	2.5
1M	–	1.8	1.8	1.9	–
2	1.8	1.9	2.1	2.2	3.5
3	3.6	3.6	3.6	3.7	4.1
4	2.3	2.3	2.7	4.2	8.5

*For population 2, this error percentage is 70.

The results for sample size 200 are not discussed in detail because they parallel those for sample size 100.

Reliability of Nominal Confidence Coefficient. Tables 5.5 through 5.8 (pp. 61–63) present the experimental results on the proportion of correct confidence intervals for the combined estimators, for the two-sided, nominal 95.4% confidence interval and the one-sided lower, nominal 93.3% confidence interval. The principal findings are the following:

1. The mpu-difference and mpu-ratio estimators generally tend to behave quite similarly. Occasional differences are minor in the context of overall similarity of behavior.

2. For all of the low error rate study populations (.5% and 1%), the proportions of correct intervals are far closer to the nominal confidence coefficients for the combined estimators than for the difference and ratio estimators alone.

3. The divergences of the proportion of correct intervals from the nominal confidence coefficient for the low error rate study populations are smallest for the least skewed populations (2 and 4) and greatest for the most highly skewed populations (1 and 3). Indeed, for populations 2 and 4, the divergences for the two-sided, nominal 95.4% interval do not exceed 1.7 percent points for sample size 100 and are even smaller for sample size 200. The results for the one-sided lower, nominal 93.3% interval are quite similar. The main distinction is that the proportions of correct intervals exceed the nominal confidence coefficient for the one-sided interval while they tend to fall below the nominal confidence coefficient for the two-sided interval.

For the low error rate study populations for the most highly skewed populations (1 and 3), the proportions of correct two-sided intervals for the combined estimators, while higher than for the individual difference and ratio estimators, still fall substantially short of the nominal 95.4% confidence coefficient. For the one-sided confidence interval, in contrast, the proportions of correct intervals are materially higher than the nominal confidence coefficient.

This behavior for the low error rate study populations, where the population skewness is associated with the divergence between the actual proportion of correct intervals and the nominal confidence coefficient, is in correspondence with the behavior of the mean-per-unit estimator.

4. For the higher error rate study populations, both skewness of the population and magnitude of errors are associated with the divergence between the actual proportion of correct intervals and the nominal confidence coefficient. The proportions of correct two-sided intervals are again far below the nominal 95.4% confidence level when the study populations with error rates of 5% or more for the highly skewed populations 1 and 3 are sampled. On the other hand, the proportions of correct two-sided intervals are within 1.6 percent points of the nominal

Table 5.5
Actual Percent of Correct Confidence
Intervals With MPU-Difference Estimator, $n = 100$

Population	Population Error Percentage				
	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	82.3	82.0	–	83.3	84.3
1M	–	90.0	90.2	89.3	–
2	93.8	94.2	94.3	94.5	93.8
3	82.7	82.5	83.0	82.8	82.3
4	93.7	94.0	94.7	94.8	78.7
	One-sided Lower, Nominal 93.3%				
1	99.2	99.2	–	99.2	99.3
1M	–	98.2	97.7	98.0	–
2	96.5	96.7	96.0	95.5	95.7
3	98.0	98.0	97.8	98.0	98.0
4	95.8	96.0	94.5	86.3	72.8

*For population 2, this error percentage is 70.

Table 5.6
Actual Percent of Correct Confidence
Intervals With MPU-Difference Estimator, $n = 200$

Population	.5	1	5	10	30*
	Population Error Percentage				
	Two-sided, Nominal 95.4%				
1	87.2	87.0	–	86.2	87.7
1M	–	92.0	92.8	91.5	–
2	95.3	95.2	95.2	96.7	95.2
3	87.5	87.7	87.3	87.5	87.8
4	96.0	96.0	96.0	92.8	88.2
	One-sided Lower, Nominal 93.3%				
1	98.0	98.3	–	98.5	98.2
1M	–	97.5	97.5	97.7	–
2	94.8	95.0	95.3	96.2	97.0
3	97.2	97.2	96.8	96.5	97.3
4	94.7	94.5	93.0	85.5	83.8

*For population 2, this error percentage is 70.

Table 5.7
Actual Percent of Correct Confidence
Intervals With MPU-Ratio Estimator, $n = 100$

Population	.5	1	5	10	30*
	Population Error Percentage				
	Two-sided, Nominal 95.4%				
1	82.3	82.0	–	83.8	85.0
1M	–	90.0	90.5	89.7	–
2	93.8	93.8	94.2	94.5	93.8
3	82.7	82.5	83.0	82.8	82.5
4	93.7	94.0	94.7	93.7	81.8
	One-sided Lower, Nominal 93.3%				
1	99.2	99.2	–	99.2	99.3
1M	–	98.2	97.7	98.0	–
2	96.5	96.7	96.0	95.5	94.8
3	98.0	98.0	97.8	98.0	98.0
4	95.8	96.2	95.0	87.8	78.5

*For population 2, this error percentage is 70.

confidence coefficient for the moderately skewed population 2, for both sample sizes. For population 4, however, which is also moderately skewed but where the error amounts are very large, the divergences between the actual proportion of correct two-sided intervals and the nominal 95.4% confidence coefficient are small only for the 5% and 10% error rate study populations. For the 30% error rate study population, the divergence is large for sample size 100 and smaller, but still to be reckoned with, for sample size 200.

When the one-sided lower, nominal 93.3% confidence interval is employed with the higher error rate study populations, the proportion of correct intervals again tends to exceed the nominal confidence coefficient—more so for the highly skewed populations than for the moderately skewed populations, and more so for sample size 100 than for sample size 200. For population 4, however, where the errors are large, the proportions of correct one-sided lower intervals are below the nominal confidence level for both the 10% and 30% error rate study populations, for both sample sizes. This divergence is particularly substantial for the 30% error rate study population and sample size 100.

The results for the two-sided, nominal 98.8% interval and the one-sided lower, nominal 97.7% interval are not discussed here because they parallel those for the two confidence intervals considered above.

Table 5.8
Actual Percent of Correct Confidence
Intervals With MPU-Ratio Estimator, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	87.2	87.0	–	86.3	88.2
1M	–	92.0	93.0	91.5	–
2	95.2	95.2	95.3	96.5	95.0
3	87.5	87.7	87.3	87.7	87.8
4	96.0	95.8	95.5	93.3	90.3
	One-sided Lower, Nominal 93.3%				
1	98.0	98.3	–	98.5	98.2
1M	–	97.5	97.5	97.5	–
2	94.8	95.0	95.3	95.8	95.0
3	97.2	97.2	96.8	96.5	97.3
4	94.5	94.5	92.8	87.8	86.8

*For population 2, this error percentage is 70.

6

Poststratified Mean-Per-Unit Estimator

An auditor may employ a simple random sample of audit units expecting to utilize a ratio or difference estimator. Should the error characteristics and error rate turn out to make it undesirable to utilize either of these estimators, the auditor might instead rely on the mean-per-unit estimator or on the mpu-difference or mpu-ratio estimators. A disadvantage of the mean-per-unit estimator is that it is frequently highly imprecise. This chapter reports on an investigation on whether the performance of the mean-per-unit estimator can be improved materially by stratification *after* the simple random sample of audit units has been selected.

Poststratification

The term "poststratification" refers to stratification employed after a simple random selection of sample units. Thus, the auditor may select a simple random sample of 100 accounts receivable and then sort these into, say, seven groups or strata according to book amount. For instance, the first stratum might include all sample accounts with balances under $25. The known numbers of accounts in each stratum are then used to estimate the population total audit value.

The mean-per-unit estimator with such a poststratification scheme (denoted a ps-mpu estimator) is developed as follows. A simple random sample of n audit units is selected. These are divided into L strata, and for each stratum the sample mean audit value is obtained—namely, $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_L$. A typical stratum mean is denoted $\bar{x}_h$. To use the ps-mpu estimator, the auditor must know the number of units in each stratum. Thus, if the strata are defined by the amount of book balance, the auditor must know the number of audit units in each stratum of book balances for the population. The number of units in a stratum for the population is denoted N_h, while the number of sample units that fall into this stratum is denoted n_h. The total population size N then is given by:

$$N = \sum_{h=1}^{L} N_h \tag{6.1}$$

and the total sample size n by:

$$n = \sum_{h=1}^{L} n_h \tag{6.2}$$

The ps-mpu estimator then can be expressed as follows:

$$\hat{X} = \sum_{h=1}^{L} N_h \bar{x}_h \tag{6.3}$$

This estimator is, in fact, a type of ratio estimator because the sample means $\bar{x}_h$ are ratio estimators here. The reason is that the denominator of the sample mean (that is, the sample size n_h) varies from sample to sample and is thus a random variable. It can be readily appreciated that n_h varies from sample to sample by recognizing that a simple random sample of n audit units has been selected. Thus, only the total sample size n is fixed. Sometimes such a sample will happen to contain a few, say, large accounts while at other times the sample will contain more large accounts.

Since the ps-mpu estimator is a type of ratio estimator, it is biased, but the bias is small when the strata sample sizes n_h are large.

The estimated variance of the ps-mpu estimator is taken to be:[1]

$$(6.4) \qquad s^2(\hat{X}) = \sum_{h=1}^{L} N_h^2\left(1 - \frac{n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

where s_h^2 is the sample variance of the audit values from the h^{th} stratum defined according to formula (3.4).

Choice of Number of Strata

The number of strata utilized may substantially affect the precision of the ps-mpu estimator. It may also affect the bias of the estimator. For any given sample size n, the larger the number of strata the smaller will be the expected strata sample sizes. If these strata sample sizes become quite small, the bias of the ratio estimator may become pronounced.

To investigate these points, as well as to note the effect of the number of strata on the proportion of correct confidence intervals, a preliminary study was undertaken for populations 1 and 1M. The basic results are presented in Appendix Tables A-55 through A-58 and A-65 through A-70 (pp. 180–187). Seven, 10, and 15 strata were investigated. Table 6.1 (p. 68) summarizes the results on bias for various study populations, presenting data on the mean of the 600 sample estimates $\hat{X}$ as a percentage of the population total audit value, for sample size 100. It is clear that there is substantial bias for 10 and 15 strata; all differences between the mean of the 600 estimates and the true total audit value far exceed sampling variation (at 95% confidence level). On the other hand, there is no indicated bias for 7 strata since all differences are within sampling variation.

Table 6.2 (p. 68) contains data for various study populations on the relative precision achieved with different numbers of strata, for sample size 100. For both populations 1 and 1M, the relative precision improves substantially as the number of strata is increased from 7 to 10. The same situation holds for population 1 as the number of strata is increased from 10 to 15.

1. This estimated variance is conditional on the observed number of observations n_h from the several strata. An alternative formula uses the finite population correction $1 - n/N$ to reflect the common sample selection probabilities in all strata. Still another alternative formula allows for the fact that the number of observations n_h from the several strata vary from sample to sample; see, for example, Leslie Kish, *Survey Sampling* (New York: John Wiley and Sons, Inc., 1965), p. 90. For reasonably large sample sizes, all three formulas provide similar results.

Table 6.1
Mean Value of 600 Estimates as Percent of
Population Total Audit Value, $n = 100$

Number of Strata	Population Error Percentage				
	.5	1	5	10	30
	Population 1				
7	100.3	100.3	–	100.3	101.8
10	92.5	92.5	–	92.5	93.9
15	–	–	–	–	91.0
	Population 1M				
7	–	100.9	100.9	100.9	–
10	–	95.2	95.3	95.3	–

This improvement in precision with more strata is not, however, accompanied by a similar improvement in the reliability of the nominal confidence coefficient. Table 6.3, opposite, summarizes the results on the proportion of correct intervals for sample size 100 for population 1. The results for population 1M follow the same pattern. As the number of strata is increased, the proportion of correct intervals tends to depart further from the nominal confidence coefficient, for both the two-sided and one-sided confidence intervals. This greater departure from the nominal confidence coefficient is especially pronounced for the two-sided confidence interval.

On the basis of these results (and those for sample size 200, which

Table 6.2
Relative Standard Error of Estimator, $n = 100$
(in percent)

Number of Strata	Population Error Percentage				
	.5	1	5	10	30
	Population 1				
7	22.8	22.8	–	22.8	26.4
10	13.5	13.5	–	13.5	17.5
15	–	–	–	–	11.9
	Population 1M				
7	–	12.1	12.0	12.1	–
10	–	6.5	6.5	6.6	–

correspond to the results already discussed), it was decided to use 7 strata in the study of the ps-mpu estimator for all populations. The 7 strata were defined for each population according to book value of the audit units so that: (1) for most strata, the expected number of sample observations for that stratum would be about 15 or more for either sample size 100 or 200 and (2) in no case would the expected number of observations for a stratum be less than about 9 for sample size 100 or about 18 for sample size 200.

Table 6.3
Actual Percent of Correct Confidence
Intervals for Population 1, $n = 100$

	Population Error Percentage				
Number of Strata	.5	1	5	10	30
	Two-sided, Nominal 95.4%				
7	74.8	74.8	–	74.7	73.2
10	50.8	50.8	–	51.8	52.5
15	–	–	–	–	46.0
	One-sided Lower, Nominal 93.3%				
7	98.8	98.8	–	99.0	98.7
10	99.3	99.3	–	99.3	98.5
15	–	–	–	–	100.0

Experimental Findings

The basic experimental results are presented in Appendix Tables A-55 through A-64 (pp. 180–184). Results were not obtained for most of the higher error rate study populations because poststratification employing the mean-per-unit estimator is most likely to be considered when the population error rate is low.

Bias of PS-MPU Estimator. Table 6.4 (p. 70) summarizes the results on bias for each study population. It presents data on the mean of the 600 sample estimates as a percentage of the true total audit value, for sample size 100. The mean values for population 3 differ by more than sampling variation (95% confidence coefficient) from the true total audit values. All other results do not differ by more than sampling variation. In any case, Table 6.4 suggests that bias, if present, is not of a major magnitude except for population 3, where the bias may amount to about 2.5% for sample size 100. Results for sample size 200 are not explicitly summarized here, but do show that the bias tends to be still smaller for that sample size.

Table 6.4
Mean Value of 600 Estimates as Percent of
Population Total Audit Value, $n = 100$

Population	Population Error Percentage .5	1	5	10	30*
1	100.3	100.3	–	100.3	101.8
1M	–	100.9	100.9	100.9	–
2	100.3	100.3	100.3	–	–
3	97.5	97.5	97.5	–	–
4	99.6	99.7	99.7	–	–

*For population 2, this error percentage is 70.

Precision of PS-MPU Estimator. Table 6.5, below, presents data on relative precision, for sample sizes 100 and 200. A number of interesting findings are conspicuous as follows:

1. Compared to the precision of the mean-per-unit estimator (Table 3.1, p. 37), that of the ps-mpu estimator tends to be somewhat better. The

Table 6.5
Relative Standard Error of Estimator
(in percent)

Population	Population Error Percentage .5	1	5	10	30*
			$n = 100$		
1	22.8	22.8	–	22.8	26.4
1M	–	12.1	12.0	12.1	–
2	8.5	8.5	8.6	–	–
3	27.8	27.8	27.8	–	–
4	11.8	11.8	12.0	–	–
			$n = 200$		
1	18.1	18.1	–	18.0	18.0
1M	–	8.5	8.5	8.5	–
2	5.9	5.9	6.0	–	–
3	20.1	20.1	20.2	–	–
4	8.1	8.2	8.3	–	–

*For population 2, this error percentage is 70.

greatest relative improvements are found for populations 2 and 4, the populations with the least skewness.

2. The ps-mpu estimator has greatest relative precision for populations 1M, 2, and 4. This corresponds with the results for the mean-per-unit estimator.

3. Compared to the difference and ratio estimators (Tables 4.2 and 4.3, pp. 44 and 45) and to the mpu-difference and mpu-ratio estimators (Tables 5.3 and 5.4, pp. 59 and 60) for the same sample size, the ps-mpu estimator is much more imprecise.

4. The relative standard error of the ps-mpu estimator tends to be stable as the error rate increases for the study populations under consideration, as is the case for the mean-per-unit estimator.

Reliability of Nominal Confidence Coefficient. Tables 6.6, below, and 6.7 (p. 72) present the experimental findings on the proportion of correct confidence intervals for the ps-mpu estimator, for the two-sided, nominal

Table 6.6
Actual Percent of Correct Confidence Intervals With Poststratified Mean-Per-Unit Estimator, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	74.8	74.8	–	74.7	73.2
1M	–	82.3	82.0	81.8	–
2	89.2	89.0	89.5	–	–
3	76.2	76.2	76.2	–	–
4	91.8	91.7	92.2	–	–
	One-sided Lower, Nominal 93.3%				
1	98.8	98.8	–	99.0	98.7
1M	–	97.3	97.3	97.3	–
2	95.5	95.5	95.3	–	–
3	98.2	98.2	98.0	–	–
4	96.0	96.2	96.0	–	–

*For population 2, this error percentage is 70.

Table 6.7
Actual Percent of Correct Confidence
Intervals With Poststratified Mean-Per-Unit
Estimator, $n = 200$

Population	Population Error Percentage .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	80.5	80.8	–	80.8	81.5
1M	–	87.5	87.7	87.2	–
2	91.3	90.8	90.7	–	–
3	85.8	85.8	85.8	–	–
4	93.2	93.0	92.8	–	–
	One-sided Lower, Nominal 93.3%				
1	98.7	98.7	–	98.5	98.3
1M	–	97.2	97.0	96.8	–
2	94.8	94.8	94.5	–	–
3	96.5	96.5	96.5	–	–
4	94.3	94.2	94.0	–	–

*For population 2, this error percentage is 70.

95.4% confidence interval and for the one-sided lower, nominal 93.3% confidence interval. Highlights of these tables are as follows:

1. The proportions of correct two-sided confidence intervals are consistently below the nominal 95.4% level.
2. The divergences between the actual proportion of correct two-sided intervals and the nominal 95.4% confidence coefficient are larger for the ps-mpu estimator than for the mpu-difference and mpu-ratio estimators (Tables 5.5 through 5.8, pp. 61–63).
3. These divergences are also larger for the ps-mpu estimator than for the mean-per-unit estimator (Tables 3.2 and 3.3, pp. 38 and 39).
4. The proportions of correct two-sided intervals are nearest to the nominal 95.4% level for populations 2 and 4, the two populations with the least skewness. For population 4, in particular, the maximum divergence is only 3.7 percent points for sample size 100 and 2.6 percent points for sample size 200. For populations 1 and 3 (the two most

skewed populations), on the other hand, the proportions of correct two-sided intervals are far below the nominal 95.4% level for the ps-mpu estimator. These results—that the reliability of the nominal confidence coefficient for the two-sided interval is greatest for the least skewed populations and is smallest for the most highly skewed populations—parallel those for the mean-per-unit estimator.

5. The proportions of correct one-sided lower confidence intervals are above the nominal 93.3% level for all study populations for which results have been obtained. Again, the divergences are smallest for populations 2 and 4. The tendency for the proportion of correct one-sided intervals to be above the nominal 93.3% level was also noted for the mean-per-unit estimator and, for lower error rate study populations, for the mpu-difference and mpu-ratio estimators.

The results for the two-sided, nominal 98.8% interval and for the one-sided lower, nominal 97.7% interval are not considered in detail here. They do parallel the results for the two confidence intervals just discussed.

Part III

Stratified Random Sampling of Audit Units

7

Stratified Mean-Per-Unit Estimator

Stratified Random Sampling of Audit Units

With stratified random sampling of audit units, the population is first divided into a number of mutually exclusive groups or strata. The basis of this division often is the book amount of the audit unit. Thus, one stratum may consist of all audit units with book amounts under $100 while another stratum may include all audit units with book amounts of $100 to $250, and so on. Stratification by book amount is helpful when the book amounts of the audit units are related to their audit values.

After the population of audit units has been divided into a number of mutually exclusive strata, a simple random sample (without replacement) of audit units is selected from each stratum. These samples must be independent, which can be accomplished by use of different sets of random numbers for the selection of the different samples. Once the various strata sample results have been obtained, they are combined to yield an estimate of the population total audit value.

The distinction between stratified random sampling of audit units and poststratification (discussed in chapter six) lies in the selection scheme for the sample. With stratified random sampling, independent random samples, each of fixed size, are selected from the various population strata. With poststratification, on the other hand, a single random sample of fixed size is selected from the entire population, and the sample is then subdivided into strata.

Mean-Per-Unit Estimator

The number of strata into which the population is divided for purposes of stratified sampling is denoted by L. The number of audit units in the h^{th} stratum of the population is denoted by N_h. A simple random sample of n_h audit units is selected from the N_h audit units in the h^{th} stratum, and the audit values for these observations are denoted $x_{h1}, x_{h2}, \ldots, x_{hn_h}$. The mean and variance for the sample audit values from the h^{th} stratum are:

$$\text{(7.1)} \qquad \bar{x}_h = \frac{\sum_{i=1}^{n_h} x_{hi}}{n_h}$$

$$\text{(7.2)} \qquad s_h^2 = \frac{\sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2}{n_h - 1}$$

The mean-per-unit estimator for the population total audit value with stratified random sampling then is:

$$\text{(7.3)} \qquad \hat{X} = \sum_{h=1}^{L} N_h \bar{x}_h$$

and the estimated variance of this estimator is:

$$\text{(7.4)} \qquad s^2(\hat{X}) = \sum_{h=1}^{L} N_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{s_h^2}{n_h}$$

The square root of this estimated variance is the estimated standard error of the mean-per-unit estimator.

Choice of Strata Boundaries

When the population error rate is very low and the population book values are known, the auditor has very effective information for stratified sample design planning. Use of stratification here differs from the usual case, in that a large number of strata are highly effective and one can also employ optimal strata boundaries.

Statistical theory provides strata boundaries for book values which are optimal when there are no errors in the population—that is, the boundaries for that case minimize the standard error of the mean-per-unit estimator for given sample size and given number of strata. The determination of these optimal boundaries unfortunately requires tedious calculations. An easier calculation is possible if a uniform distribution of items within a stratum can be assumed.[1] Generally, this assumption is quite reasonable when the number of strata is large. A computer program was developed to implement this approximate method in this empirical study, since even the approximate method of obtaining optimum boundaries is still fairly time-consuming.

The algorithm used in the computer program to obtain the optimum boundaries for stratification with the approximate method will be explained in terms of developing the optimal strata boundaries for population 1 as follows:

1. The population is divided into 100 classes of equal width and the number of audit units in each class is determined. For population 1, the minimum book value is \$.50 and the maximum book value is \$6,869.70 (Table 2.1, p. 12). Hence, the first class includes freight accounts with book amounts between \$.50 and \$69.19, and so forth.
2. If any class contains more than 3% of the total number of audit units in the population, it is further subdivided into five classes of equal width. This subdivision process is carried on up to three times should any subdivision class contain more than 3% of the total number of audit units in the population. Table 7.1 (p. 80) shows, in columns 1 and 2, the classes and frequencies obtained by this process for population 1. Altogether, 168 classes were created.
3. The width of any class is chosen as a standard or unit width and the widths of all other classes are expressed as a multiple of this standard width. In Table 7.1, the standard width is taken to be \$68.692, and the widths of all classes are expressed as a multiple of this standard width, as shown in column 3.
4. The class frequency and the class width in units of the standard width are multiplied. This is done in column 4 of Table 7.1.
5. The square root of the product is taken. This is shown in column 5 of Table 7.1.

1. See William G. Cochran, *Sampling Techniques,* 2nd ed. (New York: John Wiley & Sons, Inc., 1963), p. 130.

Table 7.1
Development of Optimal Strata Boundaries
for Population 1

Class	(1) Limits Lower	Upper	(2) Frequency	(3) Width in Standard Units	(4) (2) times (3)	(5) Square Root of (4)	(6) Cumulative of (5)
1	.50	3.25	85	.04	3.4000	1.8439	1.8439
2	3.25	6.00	146	.04	5.8400	2.4166	4.2605
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9	9.29	9.84	174	.008	1.3920	1.1798	10.4573
10	9.84	10.39	207	.008	1.6560	1.2869	11.7441
11	10.39	10.94	167	.008	1.3360	1.1559	12.9000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	15.34	15.89	157	.008	1.2560	1.1207	22.9881
21	15.89	16.44	160	.008	1.2800	1.1314	24.1194
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
168	6801.01	6869.70	1	1.0	1.0000	1.0000	175.9376
		Total	8,309				

Stratum	Limit Lower	Upper	Frequency
1	.50	10.39	1,147
2	10.39	15.89	1,582
etc.	etc.	etc.	etc.

6. The square roots are cumulated. These are shown in column 6 of Table 7.1.

7. The total of the cumulative square roots (175.9376 in Table 7.1) is divided by the desired number of strata. The resulting number represents the interval, on the cumulative square root scale, of the optimal strata. Regarding the example in Table 7.1, suppose that 15 strata are desired. First, 175.9376 is divided by 15 to yield 11.7292. In column 6, 11.7292 is not found exactly and the algorithm calls for the closest value, which here is 11.7441. Hence, the first stratum consists of all accounts with book values between $.50 and $10.39. Next, the interval 11.7292 on the cumulative square root scale is multiplied by 2, which yields 23.4583. One, therefore, needs to search in column 6 for the value closest to 23.4583, and this defines the upper boundary of the second stratum. From column 6, it is seen that the closest value is 22.9881 so that the upper boundary of the second stratum is $15.89. Other optimal boundary values are obtained by continuing this procedure.

8. The computer program then determines for each stratum the number of audit units N_h in that stratum as well as the variance of the N_h book values in that stratum, denoted S_h^2:

$$(7.5) \qquad S_h^2 = \frac{\sum_{i=1}^{N_h} (Y_{hi} - \bar{Y}_h)^2}{n_h - 1}$$

The square root of this stratum variance, denoted S_h, is the stratum standard deviation of book values.

Once the optimum strata boundaries have been obtained, and knowing the strata sizes N_h and the strata standard deviations S_h, the optimal allocation of the total sample size n to the strata is obtained from the following relation:

$$(7.6) \qquad n_h = \frac{N_h S_h}{\sum_{h=1}^{L} N_h S_h} n$$

The computer program used to calculate the optimal strata sample sizes also imposes the restriction that all strata sample sizes n_h be no smaller than 2.

At this point, the true variance of the mean-per-unit estimator in formula (7.3) can be obtained by statistical theory for the case that there are no errors in the population. This variance, denoted $S^2(\hat{X})$, is:

$$(7.7) \qquad S^2(\hat{X}) = \sum_{h=1}^{L} N_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{S_h^2}{n_h}$$

The strata boundaries obtained by the approximate procedure described above and the strata sample sizes obtained by formula (7.6) are near optimal in the sense of leading close to the smallest standard error of the mean-per-unit estimator for a given number of strata and a given total sample size, *when there are no errors in the population.*

Choice of Number of Strata

The effect of the number of strata on the standard error of the mean-per-unit estimator when there are no errors in the population can be studied by evaluating formula (7.7) for different numbers of strata, for a given sample size. Table 7.2 (p. 82) presents for 15 and 20 strata (also for 10

Table 7.2
Standard Error of Mean-Per-Unit Estimator
With 10, 15, and 20 Strata When No Errors
in Population, $n = 100$

Number of Strata	Population 1	2	3	4
10	6,438	–	–	–
15	4,260	–	–	–
Ratio	.66	–	–	–
15	4,260	34,413	163,095	68,111
20	3,237	25,199	121,094	50,977
Ratio	.76	.73	.74	.75
10	6,438	–	–	–
20	3,237	–	–	–
Ratio	.50	–	–	–

strata for population 1) the standard errors of the mean-per-unit estimator when there are no errors in the population, for all four populations and sample size 100. The results for sample size 200 are entirely similar. Table 7.2 clearly indicates that the standard error of the estimator decreases substantially with increases in the number of strata. The magnitude of this gain in precision, considering the number of strata involved, is unique for the case of no errors in the population. When errors are present and the auditor uses book values for stratification, an increase in the number of strata improves the precision only up to a point, and little further gain in increasing the precision is achieved with additional strata. William G. Cochran has conducted an analysis where the variable used for stratification and the variable of interest are only partially correlated.[2] This analysis suggests that ordinarily under these circumstances little further gain in improving precision is to be obtained beyond about 6 strata.

There is another interesting feature of Table 7.2. For each population, the reduction in the standard error is almost exactly inversely proportional to the increase in the number of strata. This is indeed the theoretical relation for the standard error with optimal boundaries and optimal allocation when the population is uniform. Cochran has found the same relation to apply approximately for a number of skewed populations.[3] The results for the

2. Cochran, Sampling Techniques, p. 134.
3. *Ibid.*, p. 133.

four populations under study, which include highly skewed populations, are certainly consistent with these earlier results.

All of these results about the effect of the number of strata on the precision of the mean-per-unit estimator apply for the case of no errors in the population. To study the effect of the number of strata on the precision of the estimator when errors are indeed present in the population, the experimentation with stratified sampling was conducted for both 15 and 20 strata.

Experimental Findings

The basic experimental results are presented in Appendix Tables A-71 through A-86 (pp. 188–195). These were obtained by using the optimal strata boundaries and the optimal strata sample sizes as determined from the book values of the audit units, when sampling the different error rate study populations. Thus, the strata boundaries and strata sample sizes employed are no longer "optimal" in these cases because the book values are not identical to the audit values when errors are present. The auditor cannot, of course, obtain "optimal" strata boundaries and strata sample sizes when errors are present, since he does not know the audit values prior to sampling. If he did, sampling would be unnecessary. Of necessity, therefore, the auditor can only utilize book values in planning the design of a stratified sample.[4] This study, based on strata boundaries and strata sample sizes which are optimal when no errors are present in the population, therefore will show the effectiveness of such planning when indeed errors do exist in the population.

Unbiasedness of Mean-Per-Unit Estimator. Statistical theory shows that the mean-per-unit estimator with stratified random sampling is unbiased. All of this study's results are in accord with this theoretical finding. In every case, the mean of the 600 sample estimates is very close to the true total audit value and, with one exception, it is well within sampling variation (95% confidence level).

Precision of Mean-Per-Unit Estimator. Table 7.3 (p. 84) presents the results on relative precision of the mean-per-unit estimator for stratified sampling of audit units with sample size 100, when the strata boundaries and strata sample sizes are based on book values but errors are actually present. Table 7.3 contains the results of both 15 and 20 strata and, to

4. The auditor may have information about population areas where errors are more likely or where the errors are likely to be larger, and can, of course, use this information for stratification.

Table 7.3
Relative Standard Error of Mean-Per-Unit
Estimator, $n = 100$
(in percent)

Population	Population Error Percentage 0	.5	1	5	10	30*
			15 Strata			
1	1.1	1.1	1.1	1.2	1.3	1.5
2	1.0	1.0	1.0	1.2	1.4	2.5
3	1.2	1.2	1.2	1.2	1.2	1.3
4	.9	1.1	1.2	1.4	2.0	4.1
			20 Strata			
1	.9	.9	.9	1.1	1.2	1.4
2	.7	.7	.8	1.1	1.3	2.5
3	.9	.9	.9	.9	1.0	1.1
4	.7	.9	.9	1.2	1.9	3.9

*For population 2, this error percentage is 70.

facilitate comparisons, presents the relative precisions for the case of no errors in the population. The following are key findings from this table:

1. The relative precisions of the mean-per-unit estimator with stratified sampling are far better than those of the mean-per-unit estimator with simple random sampling of the same sample size (Table 3.1, p. 37).

2. The mean-per-unit estimator with stratified sampling sometimes performs as well or better, in terms of relative precision, than the difference and ratio estimators with simple random sampling but at other times tends to have larger relative standard errors (Tables 4.2 and 4.3, pp. 44 and 45). Specifically, the difference and ratio estimators with simple random sampling are more precise than the mean-per-unit estimator with stratified random sampling for populations 1 and 3—the two populations having the smallest error amounts. Even here, however, the relative standard error for the stratified mean-per-unit estimator does not exceed 1.4% for population 1 and 1.1% for population 3 when 20 strata are used, compared to .9% and .4% relative standard errors, respectively, for the difference estimator with simple random sampling.

   For populations 2 and 4, the two populations for which the error amounts are largest, the situation differs substantially. For population

2, the difference and ratio estimators with simple random sampling are more precise than the stratified mean-per-unit estimator only for the lowest error rate study populations. These are the same study populations for which many simple random samples yield an estimated standard error of zero for the difference and ratio estimators. For higher error rate study populations, the precision of the mean-per-unit estimator based on stratified random sampling with 20 strata is either about the same or superior. For population 4, the mean-per-unit estimator based on stratified random sampling with 20 strata is consistently more precise than the difference and ratio estimators with simple random sampling for all study populations, particularly so for the higher error rate study populations.

3. The mean-per-unit estimator based on stratified random sampling is consistently more precise, often substantially so, then either the mpu-difference or mpu-ratio estimators based on simple random sampling (Tables 5.3 and 5.4, pp. 59 and 60).

4. The relative standard errors of the mean-per-unit estimator with stratified random sampling, for the same sample size and number of strata, tend to be at about the same level for all four populations when the error rate is 5% or less. This is in sharp contrast to the results for simple random sampling. There, the mean-per-unit estimator is much less precise for populations 1 and 3, and the difference and ratio estimators are much less precise for population 4. For the higher error rate study populations, the relative standard errors for the stratified mean-per-unit estimator do diverge more for the different populations. Still, they do not differ nearly as much as those for the difference and ratio estimators based on simple random sampling.

5. As the error rate in the population increases, the relative standard error of the stratified mean-per-unit estimator tends to increase, most rapidly for populations 2 and 4 (the two populations having the largest error amounts). Still the increases are relatively much smaller than for the difference and ratio estimators based on simple random sampling. For population 4, for instance, the relative standard error for the difference estimator is 8.4 times as large when the error rate is 30% than when it is .5%. For the stratified mean-per-unit estimator, on the other hand, the corresponding multiple is only 4.3 when 20 strata are employed and 3.7 when 15 strata are employed.

6. When the error rate is 5% or less, the relative standard error based on no errors in the population provides a reasonably close indication of the order of magnitude of the actual relative standard error for planning purposes, especially for populations 1, 2, and 3 with 15 strata. For larger error rates, the divergence between the actual relative

standard error and that for the case of no errors in the population becomes larger.

7. Increasing the number of strata from 15 to 20 leads to a smaller relative standard error. As the error rate increases, however, the improvement in relative precision by increasing the number of strata becomes smaller. In fact, when the error rate is at the highest level, the relative standard errors for 15 and 20 strata are quite close to each other for most of the populations.

8. When errors are present in the population, the precision for the case of no errors is a somewhat poorer guide for planning a stratified sample with 20 strata than for planning one with 15 strata. Table 7.4, below, demonstrates this point, showing the actual standard error as a percentage of the standard error for the case of no errors in the population, for sample size 100 with each of the four populations. In general, the actual standard error departs relatively more from the standard error for no errors when 20 rather than 15 strata are used.

The results for sample size 200 are not discussed in detail, but they correspond closely to those for sample size 100.

Table 7.4
Standard Error of Mean-Per-Unit Estimator
as a Percent of Standard Error When No Errors
in Population, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
			15 Strata		
1	100	100	107	112	130
2	99	103	124	143	264
3	102	102	103	104	109
4	122	127	149	211	385
			20 Strata		
1	102	104	124	136	169
2	99	105	154	181	359
3	99	100	103	108	120
4	126	135	181	272	498

* For population 2, this error percentage is 70.

Reliability of Nominal Confidence Coefficient. Tables 7.5 through 7.8 (pp. 87–90) present the experimental results on the proportion of correct confidence intervals for the mean-per-unit estimator with stratified random sampling. Results presented are for the two-sided, nominal 95.4% confidence interval and for the one-sided lower, nominal 93.3% confidence interval. The following interesting findings stand out from these tables:

1. Considering all populations and all error rates, the actual proportions of correct intervals for the stratified mean-per-unit estimator tend to be far closer to the nominal confidence coefficients than they are for any of the estimators based on simple random sampling of audit units.

2. For populations 1, 2, and 3, the proportions of correct two-sided intervals are quite close to the nominal 95.4% confidence level for all error rates. The maximum divergence for these populations, for sample size 100 and 15 strata, is 1.3 percent points. The corresponding figure for sample size 100 and 20 strata is 1.9 percent points. The results for sample size 200 are similar.

Table 7.5
Actual Percent of Correct Confidence
Intervals With Mean-Per-Unit Estimator
for 15 Strata, $n = 100$

Population	Population Error Percentage: .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	94.2	94.2	94.7	95.0	95.5
2	96.3	96.2	96.7	96.0	94.7
3	95.5	95.5	95.8	95.7	95.8
4	94.7	94.7	93.3	91.0	95.0
	One-sided Lower, Nominal 93.3%				
1	94.7	94.8	94.7	96.0	95.0
2	93.5	93.7	94.2	93.8	95.0
3	93.5	93.0	93.2	93.8	93.8
4	89.5	89.5	87.5	85.7	90.5

*For population 2, this error percentage is 70.

3. Only for population 4 do the proportions of correct two-sided intervals diverge to a greater extent from the nominal 95.4% confidence level.

The maximum divergence is 4.4 percent points for sample size 100 and 15 strata, and 6.2 percent points for the same sample size and 20 strata. For sample size 200, the maximum divergences are much smaller.

4. The proportions of correct one-sided lower confidence intervals are close to the nominal 93.3% confidence level for populations 1, 2, and 3. The maximum divergence is 2.7 percent points for sample size 100

Table 7.6
Actual Percent of Correct Confidence Intervals With Mean-Per-Unit Estimator for 15 Strata, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	96.3	95.8	95.8	97.2	96.0
2	95.3	95.8	96.2	96.3	94.3
3	94.7	94.7	94.8	96.2	96.7
4	94.8	96.2	95.7	94.2	95.0
	One-sided Lower, Nominal 93.3%				
1	94.8	94.5	95.2	96.5	94.3
2	95.0	93.8	94.0	93.7	93.2
3	93.0	93.2	94.2	93.8	94.0
4	91.7	91.7	89.5	90.3	91.0

*For population 2, this error percentage is 70.

and 15 strata, and it is 2.3 percent points for the same sample size and 20 strata. Similar results are found for sample size 200.

5. Again, the proportions of correct one-sided lower confidence intervals diverge more substantially from the nominal 93.3% confidence level for population 4. The maximum divergence is 7.6 percent points for sample size 100 and 15 strata and 10.8 percent points for the same sample size and 20 strata. The maximum divergences are smaller for sample size 200.

6. No strong tendency is apparent for the nominal confidence coefficient to be substantially more reliable with one number of strata than with

Table 7.7
Actual Percent of Correct Confidence
Intervals With Mean-Per-Unit Estimator
for 20 Strata, $n = 100$

Population	.5	1	5	10	30*
			Two-sided, Nominal 95.4%		
1	95.3	95.0	94.3	93.8	95.2
2	96.0	96.8	96.5	97.3	95.8
3	96.7	96.8	96.3	96.3	94.8
4	95.8	95.8	91.8	89.2	94.2
			One-sided Lower, Nominal 93.3%		
1	93.5	93.7	92.8	94.5	95.2
2	93.5	94.2	95.3	95.5	94.7
3	93.8	93.7	93.0	91.0	92.2
4	90.7	89.7	82.5	86.2	90.0

(Column heads: Population Error Percentage)

*For population 2, this error percentage is 70.

the other. Sometimes the actual proportion of correct intervals is closer to the nominal level for 15 strata; sometimes it is closer for 20 strata. Nor is there any consistent pattern apparent when one number of strata leads to a substantially more reliable nominal confidence coefficient than does the other number of strata.

The results for the two-sided, nominal 98.8% confidence interval and for the one-sided lower, nominal 97.7% confidence interval are not discussed explicitly because they are similar to the results for the two confidence intervals considered above.

Interestingly, Appendix Tables A-71 through A-86 (pp. 188–195) show two major distinguishing features between the characteristics of the stratified mean-per-unit estimator for population 4 and for the other three populations as follows:

1. The skewness of the distribution of the standardized statistic Z tends to be positive and modestly large for the higher error rate study populations for population 4 while it tends to be small (positive or negative) for the error rate study populations for the other populations.

2. The coefficient of correlation between $\hat{X}$ and $s(\hat{X})$ is large negative for population 4 but is positive or small negative for the other populations.

Table 7.8
Actual Percent of Correct Confidence Intervals With Mean-Per-Unit Estimator for 20 Strata, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	95.3	95.8	94.7	94.2	93.8
2	96.7	96.5	96.5	95.3	96.0
3	96.0	96.3	96.3	95.8	95.7
4	92.3	93.7	92.5	95.8	94.8
	One-sided Lower, Nominal 93.3%				
1	93.8	93.5	94.5	95.3	95.0
2	93.3	92.2	94.5	94.8	94.3
3	93.7	93.8	94.3	93.7	92.2
4	88.0	87.2	86.2	90.3	91.7

*For population 2, this error percentage is 70.

8

Stratified Difference and Ratio Estimators

When a stratified random sample of audit units is selected, a difference or a ratio estimator can be utilized for estimating the population total audit value provided the auditor has information about the book values of the audit units in the population. In this chapter, the results of the empirical study on difference and ratio estimators based on stratified random sampling of audit units are presented. The strata and sample allocations utilized are exactly the same as those used for the mean-per-unit estimator. While these strata and sample allocations minimize the standard error of the mean-per-unit estimator for a given number of strata and a given sample size when there are no errors in the population, they are not optimal for the difference and ratio estimators when the population contains errors.

Difference Estimator

As before, let $x_{h1}, x_{h2}, \ldots, x_{hn_h}$ denote the audit values of the n_h sample observations from the h^{th} stratum. Further, let $y_{h1}, y_{h2}, \ldots, y_{hn_h}$ denote their corresponding book values and $\bar{y}_h$ the mean of these book values:

$$(8.1) \qquad \bar{y}_h = \frac{\sum_{i=1}^{n_h} y_{hi}}{n_h}$$

Finally, the total book value of all audit units in the h^{th} stratum is denoted Y_h:

(8.2) $$Y_h = \sum_{i=1}^{N_h} Y_{hi}$$

The difference estimator of the population total audit value with stratified random sampling then is:

(8.3) $$\hat{X} = \sum_{h=1}^{L} [Y_h + N_h(\bar{x}_h - \bar{y}_h)]$$

When no errors are found in the sample, the difference estimate will equal Y, the population total book value.

The estimated variance of this difference estimator is:

(8.4) $$s^2(\hat{X}) = \sum_{h=1}^{L} \left\{ N_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{1}{n_h(n_h - 1)} \left[\sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2 + \sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2 - 2 \sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)(y_{hi} - \bar{y}_h) \right] \right\}$$

This estimated variance will equal zero when there are no errors in the sample.

Ratio Estimator

The ratio estimator of the population total audit value utilized in this study is sometimes called a "combined" ratio estimator.[1] It is:

(8.5) $$\hat{X} = rY$$

---

1. See, for example, William G. Cochran, *Sampling Techniques,* 2nd ed. (New York: John Wiley & Sons, Inc., 1963), p. 169. The combined ratio estimator is to be distinguished from the "separate" ratio estimator. The latter estimator is:

$$\hat{X} = \sum_{h=1}^{L} r_h Y_h \quad \text{where} \quad r_h = \frac{\bar{x}_h}{\bar{y}_h}$$

The separate ratio estimator may be subject to relatively substantial bias when the strata sample sizes n_h are small.

where:

$$(8.6) \qquad r = \frac{\sum_{h=1}^{L} N_h \bar{x}_h}{\sum_{h=1}^{L} N_h \bar{y}_h}$$

and Y is the population total book value. Like the difference estimate, the ratio estimate will equal Y, the population total book value, when no errors are found in the sample. The ratio estimator is biased, but the bias is small when the sample size is large.

The estimated variance of the ratio estimator is:

$$(8.7) \qquad s^2(\hat{X}) = \sum_{h=1}^{L} \left\{ N_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{1}{n_h(n_h - 1)} \left[\sum_{i=1}^{n_h} x_{hi}^2 - 2r \sum_{i=1}^{n_h} x_{hi} y_{hi} + r^2 \sum_{i=1}^{n_h} y_{hi}^2 \right] \right\}$$

where r is defined in formula (8.6). This estimated variance is biased, but the bias is small for large sample sizes. When no errors are found in the sample, the estimated variance of the ratio estimator equals zero, as is the case for the difference estimator.

Experimental Findings

The basic experimental results are found in Appendix Tables A-87 through A-118 (pp. 196–227).

Bias of Ratio Estimator. Table 8.1 (p. 94) presents data on the mean of the 600 sample ratio estimates as a percentage of the true total audit value for each study population, for sample size 100 and 15 strata. There is no evidence of any substantial bias in this table for sample size 100. All but one difference are within margins of sampling variation (95% confidence level), and that difference is less than .02%. Results for sample size 200 and for 20 strata are similar. Hence, for the populations, sample sizes, and number of strata considered here, the bias of the ratio estimator with stratified sampling is negligibly small.

Precision of Difference and Ratio Estimators. Table 8.2 (p. 94) contains data on the relative standard error of the difference estimator, for sample size 100 for both 15 and 20 strata. Table 8.3 (p. 95) contains comparable

Table 8.1
Mean Value of 600 Ratio Estimates as Percent of Population Total Audit Value for 15 Strata, $n = 100$

Population	.5	1	5	10	30*
	Population Error Percentage				
1	100.0	100.0	100.0	100.0	100.0
2	100.0	100.0	100.0	100.0	100.0
3	100.0	100.0	100.0	100.0	100.0
4	100.0	100.0	100.0	99.9	99.8

* For population 2, this error percentage is 70.

data for the ratio estimator. The major findings from these tables are:

1. The relative standard errors of the difference and ratio estimators based on stratified sampling are practically the same, for all study populations.
2. Use of 15 strata leads practically to the same relative standard errors as use of 20 strata. For some study populations, indeed, the relative standard error for 15 strata may be a little smaller than that for 20 strata.

Table 8.2
Relative Standard Error of Difference Estimator, $n = 100$
(in percent)

Population	.5	1	5	10	30*
	Population Error Percentage				
			15 Strata		
1	.2	.2	.4	.7	1.1
2	.2	.3	.7	1.0	2.4
3	.1	.1	.2	.3	.6
4	.6	.7	1.0	1.8	4.0
			20 Strata		
1	.2	.2	.5	.7	1.1
2	.1	.3	.9	1.1	2.4
3	.1	.1	.2	.3	.6
4	.6	.7	1.0	1.8	3.9

* For population 2, this error percentage is 70.

Table 8.3
Relative Standard Error of Ratio
Estimator, $n = 100$
(in percent)

Population	Population Error Percentage .5	1	5	10	30*
			15 Strata		
1	.2	.2	.4	.7	1.1
2	.2	.3	.8	1.0	2.4
3	.1	.1	.2	.3	.6
4	.6	.7	1.0	1.8	4.0
			20 Strata		
1	.2	.2	.5	.7	1.1
2	.1	.3	.9	1.1	2.4
3	.1	.1	.2	.3	.6
4	.6	.7	1.0	1.8	3.9

*For population 2, this error percentage is 70.

3. Stratification tends to lead to smaller relative standard errors of the difference and ratio estimators than simple random sampling for populations 2 and 4, the two populations having the largest error amounts (see Tables 4.2 and 4.3, pp. 44 and 45). The improvement in precision is especially great for the higher error rate study populations for population 4. On the other hand, there is no gain from stratification for populations 1 and 3, the two populations having the smallest error amounts. Indeed, in some instances the relative standard errors with stratification may be larger than those for simple random sampling.

4. The difference and ratio estimators tend to be more precise than the mean-per-unit estimator for all populations studied when stratified sampling is employed (see Table 7.3, p. 84). The differences in precision are consistently greatest for populations 1 and 3. For populations 2 and 4, on the other hand, the relative standard errors of the mean-per-unit, ratio, and difference estimators are quite close to each other for the higher error rate study populations.

The results for sample size 200 are not discussed in detail, but they parallel those for sample size 100.

Percent of Samples With Errors. Table 8.4 (p. 96), shows, for each study population, the percentage of samples which contain at least one error

Table 8.4
Percent of 600 Stratified Random Samples
Containing One or More Errors, for 15 Strata

Population	.5	1	5	10	30*
			$n = 100$		
1	19.7	63.0	99.2	100.0	100.0
2	26.2	57.8	99.5	100.0	100.0
3	37.8	59.2	99.2	100.0	100.0
4	43.5	69.2	99.5	100.0	100.0
			$n = 200$		
1	40.8	91.5	100.0	100.0	100.0
2	49.7	81.5	100.0	100.0	100.0
3	64.8	87.5	100.0	100.0	100.0
4	66.5	91.3	100.0	100.0	100.0

(Column heading: Population Error Percentage)

*For population 2, this error percentage is 70.

when stratified sampling employs 15 strata. The results for stratified sampling when 20 strata are employed are entirely similar. Table 8.4 tells the expected story: When the error rate is low, a large proportion of samples (more than half when the error rate is .5% and the sample size is 100) contain no errors and thus lead to estimated standard errors of the difference and ratio estimators which equal zero.[2] When the error rate reaches 5%, this proportion becomes very small.

Reliability of Nominal Confidence Coefficient. Tables 8.5 through 8.12 (pp. 97–101) contain the experimental data on the percentage of "correct" confidence intervals based on the difference and ratio estimators with stratified random sampling, for the two-sided, nominal 95.4% confidence interval and for the one-sided lower, nominal 93.3% confidence interval. Whenever the estimated standard error of the estimator equals zero, the

2. The proportions of samples containing one or more errors are not as consistent among the populations when the error rate is low as with simple random sampling (see Table 4.4, p. 47). The reason is that the strata sample sizes are not proportionate and differ from one population to another. Also, the error rates in the several strata for a population are not equal.

confidence interval is considered incorrect. The key findings from these tables are the following:

1. The proportions of correct confidence intervals for the difference and ratio estimators based on stratified random sampling are quite similar for all populations, both sample sizes, and both 15 and 20 strata. This similarity in behavior of the two estimators was also found with simple random sampling of audit units.
2. Considering all populations and all error rates, the reliability of the nominal confidence coefficients for the stratified difference and ratio estimators tends to be not as good as for the stratified mean-per-unit estimator. It was noted in chapter seven that the actual proportions of correct intervals for the mean-per-unit estimator are close to the nominal levels for all populations except population 4, where the divergences are somewhat larger for some of the error rate study populations. Even for population 4, however, the proportions of correct confidence intervals for the stratified mean-per-unit estimator tend to be about as close as or closer to the nominal confidence levels as for the difference and ratio estimators based on stratified random sampling.

Table 8.5
Actual Percent of Correct† Confidence
Intervals With Difference Estimator for
15 Strata, $n = 100$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	18.3	55.2	96.8	92.3	95.3
2	20.5	43.0	87.2	97.2	95.5
3	8.7	14.8	48.3	67.3	86.8
4	29.5	39.7	73.0	86.2	95.2
	One-sided Lower, Nominal 93.3%				
1	19.7	63.0	95.2	98.5	96.7
2	26.2	40.3	98.5	96.2	95.7
3	8.7	14.8	47.5	63.5	83.5
4	29.5	39.5	69.3	83.2	89.7

†If estimated standard error is zero, interval is considered incorrect.
*For population 2, this error percentage is 70.

Table 8.6
Actual Percent of Correct† Confidence
Intervals With Difference Estimator for
15 Strata, $n = 200$

Population	Population Error Percentage .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	27.3	81.7	94.3	94.7	92.7
2	42.0	65.2	96.5	98.3	93.8
3	18.7	29.3	63.0	81.2	92.8
4	43.5	60.2	81.7	91.7	95.0
	One-sided Lower, Nominal 93.3%				
1	40.8	91.5	98.7	98.8	96.0
2	49.7	64.0	97.0	96.0	92.2
3	18.7	28.0	61.5	77.2	87.7
4	42.3	58.5	77.2	85.0	90.5

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.7
Actual Percent of Correct† Confidence
Intervals With Difference Estimator for
20 Strata, $n = 100$

Population	Population Error Percentage .5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	22.5	51.8	95.3	94.0	95.0
2	21.3	47.7	83.3	96.8	95.3
3	7.5	15.7	46.2	60.7	88.7
4	31.3	41.2	71.8	85.8	93.2
	One-sided Lower, Nominal 93.3%				
1	23.5	62.2	95.7	97.2	95.8
2	28.2	43.8	97.7	96.3	95.0
3	7.5	15.7	45.5	58.3	85.5
4	31.3	40.3	69.7	84.2	90.2

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.8
Actual Percent of Correct† Confidence Intervals With Difference Estimator for 20 Strata, $n = 200$

Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	25.3	78.2	94.3	92.8	92.7
2	39.2	64.7	95.0	97.5	96.0
3	17.0	29.3	66.7	80.7	92.0
4	44.8	60.8	84.2	93.3	94.0
	One-sided Lower, Nominal 93.3%				
1	44.0	90.7	97.5	97.8	96.8
2	47.2	63.5	98.8	94.5	95.5
3	17.0	27.8	64.0	77.8	88.0
4	44.5	59.3	80.2	87.5	91.2

Population Error Percentage applies to columns .5 through 30*.

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.9
Actual Percent of Correct† Confidence Intervals With Ratio Estimator for 15 Strata, $n = 100$

Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	18.3	55.5	97.7	94.5	97.5
2	21.8	44.5	89.2	97.5	96.8
3	8.7	14.8	48.3	68.5	89.8
4	29.5	41.2	74.0	88.3	96.0
	One-sided Lower, Nominal 93.3%				
1	19.7	63.0	96.7	98.8	99.0
2	26.2	40.5	99.2	98.3	97.3
3	8.7	14.8	48.0	64.5	85.5
4	29.5	39.7	70.8	84.3	91.2

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.10
Actual Percent of Correct† Confidence
Intervals With Ratio Estimator for 15
Strata, $n = 200$

Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	27.3	82.2	96.3	95.3	94.3
2	42.0	65.3	96.8	98.8	95.0
3	18.7	29.3	63.5	81.3	94.2
4	43.5	60.5	82.8	92.3	95.2
	One-sided Lower, Nominal 93.3%				
1	40.8	91.5	99.0	98.8	97.5
2	49.7	64.3	98.2	97.2	93.5
3	18.7	28.5	61.7	78.3	89.7
4	42.3	58.5	78.2	85.8	91.5

Population Error Percentage (column headings .5 to 30*)

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.11
Actual Percent of Correct† Confidence
Intervals With Ratio Estimator for 20
Strata, $n = 100$

Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	22.8	52.2	97.0	96.2	97.8
2	23.0	49.8	86.8	98.0	97.8
3	7.5	15.7	46.7	62.3	91.3
4	31.3	42.8	73.0	87.7	95.7
	One-sided Lower, Nominal 93.3%				
1	23.5	62.2	97.0	99.8	98.5
2	28.2	44.8	98.7	99.0	97.7
3	7.5	15.7	46.0	59.7	88.5
4	31.3	40.5	70.8	85.0	91.0

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Table 8.12
Actual Percent of Correct† Confidence Intervals With Ratio Estimator for 20 Strata, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	26.2	78.5	96.3	95.3	95.7
2	39.2	65.0	96.0	98.0	97.2
3	17.0	29.3	67.5	81.8	94.3
4	44.8	61.0	85.2	94.2	95.2
	One-sided Lower, Nominal 93.3%				
1	44.0	90.7	97.5	99.2	98.2
2	47.2	63.7	99.2	95.5	96.2
3	17.0	27.8	64.5	78.0	90.0
4	44.8	59.5	80.8	88.7	91.8

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

3. For low error rates (.5% and 1%), the proportions of correct confidence intervals for the stratified difference and ratio estimators based on sample size 100 are so far below the nominal confidence levels as to render the latter meaningless. Even when the sample size is 200, the proportion of correct intervals is close to the nominal level only for the 1% error rate study population for population 1, for the one-sided lower confidence interval.

4. For higher error rates, the proportions of correct intervals tend to come closer to the nominal levels, particularly for populations 1 and 2 (which have both overstatement and understatement errors). The reliability of the nominal confidence coefficients is particularly poor for the 5% and 10% error rate study populations for population 3.

5. Table 8.13 (p. 102) summarizes the magnitude of the problem of lack of reliability of the nominal confidence coefficients for the stratified difference and ratio estimators based on 15 strata. The results for 20 strata are quite similar. For purposes of this summary, the proportion of correct intervals is considered to be "close" to the nominal level if it is within 5 percent points below or anywhere above the nominal level. Table 8.13 shows the first error rate for which the actual proportion of

Table 8.13
Error Rate for Which Proportion of Correct Intervals First is Close† to Nominal Level for Difference and Ratio Estimators Based on 15 Strata

Population	$n = 100$	$n = 200$
	Two-sided, Nominal 95.4%	
1	5%	5%
2	10%	5%
3	None	30%
4	30%	10%
	One-sided Lower, Nominal 93.3%	
1	5%	1%
2	5%	5%
3	None	D-None; R-30%*
4	30%	30%

† Close is defined to be within 5 percent points below nominal level or anywhere above it.
* D: difference estimator
R: ratio estimator

correct intervals is "close" to the nominal level according to this criterion. For populations 1 and 2 (where errors are in both directions), an error rate of 5% or more tends to be necessary for the proportion of correct intervals to be "close" to the nominal level. The situation is much worse for populations 3 and 4 (where all errors are overstatements). For population 4, an error rate of 30% tends to be necessary for the proportion of correct intervals to be "close." For population 3, even the 30% error rate frequently does not yield a "close" proportion of correct intervals.

6. The proportions of correct confidence intervals for the stratified difference and ratio estimators tend to be closer to the nominal confidence levels for the higher error rate study populations for population 4 than is the case for the difference and ratio estimators based on simple random sampling. The opposite tends to be true for population 3.

7. For the populations and sample sizes considered, there does not appear to be any systematic strong effect of the number of strata on the reliability of the nominal confidence coefficients for the stratified difference and ratio estimators.

The results for the two-sided, nominal 98.8% confidence interval and for the one-sided lower, nominal 97.7% confidence interval are not discussed here, but they parallel the results for the two confidence intervals considered above.

Part IV

Dollar Unit Sampling

9

CAV Procedure

Simple Random Sampling of Dollar Units

With simple random sampling of dollar units, individual dollars are sampled at random. To illustrate, consider a population of five accounts receivable:

Account	Book Amount	Cumulative Book Amount Range
A	100	1–100
B	50	101–150
C	20	151–170
D	200	171–370
E	130	371–500
	Total 500	

This population consists of 500 dollar units. To select a random sample of, say, 3 dollar units, three random numbers from 1 to 500 are selected. Suppose the random numbers are 39, 241, and 486. The cumulative book amount column may then be used to identify the sample dollar units. Dollar unit 39 is the thirty-ninth dollar unit, in a specified sequence, of account *A*. Dollar unit 241 is the seventy-first dollar unit of account *D*, and dollar unit 486 is the one hundred-sixteenth dollar unit of account *E*.

Simple random sampling of dollar units has been called "dollar unit sampling" by Anderson and Teitlebaum.[1] While an individual dollar unit is sampled with this procedure, the auditor still must audit the entire audit unit to which the sample dollar belongs. Thereupon, an assignment of any error found in that audit unit is made to the sample dollar unit. This assignment typically is a pro rata one. Thus, if an audit of an account with a $500 book amount discloses a $100 overstatement error, the sample dollar unit from this account would be considered to have an overstatement error of $.20 with this assignment procedure.

With dollar unit sampling, it is entirely possible that several dollar units from the same audit unit are included in the random sample. If so, the affected audit unit will be audited only once, but the different sample dollar units from the same audit unit will have to be treated as separate sample units.

Dollar unit sampling was performed in this study with replacement. In view of the large number of dollar units in the study populations and the relatively small number of dollar units in the sample, sampling with replacement is practically the equivalent of sampling without replacement.

CAV Procedure

A CAV procedure leads to a bound on the population total error which is based on *combined attributes-variables* principles. Essentially, obtaining a CAV bound involves a two-step procedure. First, the number of dollars in error in the population is estimated with a one-sided upper confidence limit based on attributes theory. When only overstatement errors can occur and assuming that all such errors are 100% overstatements, the upper confidence limit for the number of dollars in error in the population is also an upper confidence limit for the total overstatement error in the population. The second step then involves reducing this upper confidence limit when overstatement errors which are less than 100% overstatements are found in the sample. The resulting modified upper confidence limit thus combines attributes and variables estimation to yield a more precise bound for the population total overstatement.

Procedures for obtaining CAV bounds have been described by Rod Anderson and A. D. Teitlebaum[2] and by Giles R. Meikle.[3] These procedures have been reviewed by James L. Goodfellow, James K. Loebbecke, and

1. Rod Anderson and A. D. Teitlebaum, "Dollar-Unit Sampling," *Canadian Chartered Accountant,* April 1973, pp. 30–39.
2. *Ibid.*
3. Giles R. Meikle, *Statistical Sampling in an Audit Context* (Toronto: The Canadian Institute of Chartered Accountants, 1972).

John Neter,[4] who have suggested another CAV bound as a candidate for consideration. The latter CAV bound was used in this study because it is simpler to understand and operationally less complex than the other CAV bounds. However, the CAV bound employed in this study may be somewhat more conservative and therefore less efficient than the other CAV bounds.

Throughout this discussion it is assumed that all errors are overstatement errors. Let Y denote, as before, the population total book value:

$$(9.1) \qquad Y = \sum_{i=1}^{N} Y_i$$

For the i^{th} sample dollar unit, let y_i denote the book amount of the audit unit to which the sample dollar belongs and let x_i denote the audit value of this audit unit. With the pro rata assignment method, the amount of the audit value for the i^{th} sample dollar unit, denoted w_i, is:

$$(9.2) \qquad w_i = \frac{x_i}{y_i}$$

We now define a variable v_i as follows:

$$(9.3) \qquad v_i = \begin{matrix} 0 \text{ if } w_i = 1 \\ 1 \text{ if } w_i \neq 1 \end{matrix}$$

Thus, $v_i = 0$ if the sample dollar unit is correct and $v_i = 1$ if the sample dollar unit is incorrect. Hence, the total number of sample dollar units in error in the sample, denoted k, is:

$$(9.4) \qquad k = \sum_{i=1}^{n} v_i$$

The CAV bound utilized in this study can then be represented as follows:[5]

$$(9.5) \qquad UCL = YP_U(k) - \frac{Y}{n} \sum_{i=1}^{n} w_i v_i$$

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4. James L. Goodfellow, James K. Loebbecke, and John Neter, "Some Perspectives on CAV Sampling Plans," Part I, *CA Magazine,* October 1974, pp. 23–30; idem, Part II *CA Magazine*, November 1974, pp. 46–53.
5. For a more complete explanation of this bound, see James L. Goodfellow *et al.*, "Some Perspectives on CAV Sampling Plans," Part II, *CA Magazine,* November 1974, pp. 51–52.

Here, UCL is the one-sided upper confidence limit for the population total overstatement, and $P_U(k)$ is the one-sided upper confidence limit (based on attributes theory) for the proportion of dollars in error in the population when k dollar units are found in error in the sample. The one-sided upper confidence limit $P_U(k)$ may be obtained from tables based on the binominal distribution or, when appropriate, on the Poisson approximation. The confidence coefficient used to obtain the upper confidence limit $P_U(k)$ is then taken as the nominal confidence coefficient for the CAV bound in formula (9.5).

To illustrate the use of this estimator, suppose that the population contains $Y = \$100{,}000$, and that a simple random sample of $n = 100$ dollar units is selected. Suppose further that all sample dollar units contain no errors except one, namely $w_{24} = .80$ and all other $w_i = 1$. Hence, all $v_i = 0$ except $v_{24} = 1$. Thus $k = 1$. For a 95% confidence coefficient, $P_U(1) = .0466$. We thus obtain as the CAV bound:

$$UCL = 100{,}000(.0466) - \frac{100{,}000}{100}(.80) = \$3{,}860$$

In previous chapters, the quantity estimated was always the population total audit value X. Indeed, the CAV one-sided upper confidence limit for the population total overstatement in formula (9.5) can be converted into a one-sided lower confidence limit for X. Let E denote the population total overstatement error, where E is defined as before:

$$(9.6) \qquad E = Y - X$$

Now, the one-sided upper confidence interval for E according to formula (9.5) is:

$$E \leq UCL$$

Rewriting E according to formula (9.6) yields:

$$Y - X \leq UCL$$

or:

$$-X \leq -Y + UCL$$

which can be rewritten:

$$(9.7) \qquad X \geq Y - UCL$$

Thus, $Y - UCL$ is a one-sided lower confidence limit for the population total audit value X.

For the previous example, where $UCL = \$3{,}860$, we would obtain as the confidence limit for the population total audit value:

$$X \geq 100{,}000 - 3{,}860 = \$96{,}140$$

Experimental Findings

The basic experimental results for the CAV bound are found in Appendix Tables A-119 through A-122 (pp. 227–229). Only populations 3 and 4 were considered because they are the only ones where all errors are overstatements.

All results in the Appendix tables pertain to the upper confidence limit *UCL* for the population total overstatement, based on a nominal 95% confidence coefficient. Each Appendix table shows first the true total overstatement for each study population. Next are shown the usual characteristics (mean, standard deviation, skewness, kurtosis) for the 600 *UCL* values obtained for each study population based on a nominal 95% confidence coefficient. Also presented are the largest and smallest of the 600 *UCL* values. Finally, each Appendix table presents data on the proportion of the 600 confidence intervals which are correct.

Reliability of Nominal Confidence Coefficient. Table 9.1, below, contains the experimental findings on the proportion of correct CAV one-sided upper, nominal 95% confidence intervals for the population total overstatement. This table clearly shows that for all study populations for populations 3 and 4, the actual proportion of correct intervals exceeds the nominal confidence

Table 9.1
Actual Percent of Correct Confidence
Intervals With Dollar Unit Sampling CAV
Bound, Nominal 95% Confidence Coefficient

Population	Population Error Percentage				
	.5	1	5	10	30
			$n = 100$		
3	100.0	–	100.0	100.0	100.0
4	100.0	–	100.0	100.0	99.3
			$n = 200$		
3	100.0	–	100.0	100.0	100.0
4	100.0	–	100.0	99.8	99.5

coefficient of 95%. In each case the difference is statistically significant (at significance level .05). Indeed, only for the large error rate study populations for population 4 are the actual proportions of correct intervals less than 100%.

Since the CAV bound is principally intended for use with low error rate populations, a comparison with the behavior of ratio and difference estimators is not useful. This is because the latter estimators perform badly, in terms of the reliability of the nominal confidence coefficient, when the population error rate is low. A comparison with the mean-per-unit estimator based on stratified sampling of audit units is more relevant. The one-sided lower, nominal 93.3% confidence interval for the population total audit value is the one most comparable to the nominal 95% confidence interval for the population total overstatement used with the CAV bound. For this one-sided interval based on the stratified mean-per-unit estimator, the proportions of correct intervals are close to the nominal 93.3% level for the study populations for population 3, but they are somewhat below the nominal confidence level for the study populations for population 4 (see Tables 7.5 through 7.8, pp. 87–90). Thus, for population 4 the nominal confidence coefficient for the one-sided interval indicates greater assurance of correct results than is warranted for the stratified mean-per-unit estimator and less assurance than is warranted for the CAV estimator.

Precision of CAV Bound. Table 9.2, below, presents, for each study population, the mean UCL value for the 600 experimental trials expressed as a relative of the true total error amount. It is evident from Table 9.2 that for the low error rate study populations, the CAV upper confidence limit tends to be far above the total error amount. As the error rate increases, on the

Table 9.2
Mean of 600 UCL Values as Relative of
Population Total Error Amount, Nominal
95% Confidence Coefficient

	Population Error Percentage				
Population	.5	1	5	10	30
			$n = 100$		
3	177.9	–	30.7	17.8	8.1
4	11.1	–	4.7	2.8	1.6
			$n = 200$		
3	100.4	–	20.1	12.1	5.8
4	6.6	–	3.3	2.2	1.4

other hand, the upper confidence limit tends to come much closer to the total error amount. Thus, the CAV bound for the total error amount tends to be relatively more precise for larger error rates for populations 3 and 4. In addition, Table 9.2 indicates that the upper confidence limits are relatively much closer to the total error amount for population 4 than for population 3. In this connection, it will be recalled that the error amounts for population 4 are much larger than those for population 3.

The relatives shown in Table 9.2 are large partly because the base is the total error amount. The perspective changes when the confidence limits are changed according to formula (9.7) to yield bounds on the total audit value (as has been the focus of discussion prior to this chapter). Table 9.3, below, presents for each study population the mean of the 600 lower confidence

Table 9.3
Mean of 600 Lower Confidence Limit
Values for Population Total Audit Value as
Percent of Total Audit Value, CAV Bound
With Nominal 95% Confidence Coefficient

	Population Error Percentage				
Population	.5	1	5	10	30
			$n = 100$		
3	96.7	-	94.8	94.0	91.5
4	96.6	-	94.9	93.4	90.4
			$n = 200$		
3	98.2	-	96.7	96.0	94.3
4	98.1	-	96.8	95.7	93.3

limits for the population total audit value with the CAV procedure at a nominal 95% confidence coefficient, expressed as a percentage of the population total audit value. From this perspective, the CAV confidence limits are more precise for the lower error rate study populations and less precise for the higher error rate study populations. This same tendency was found for the mean-per-unit estimator with stratified sampling of audit units. Table 9.3 also suggests that the relative precision for estimating the population total audit value with the CAV procedure is about the same for populations 3 and 4 when the error rate is small or moderate, and may be slightly better for population 3 than for population 4 when the error rate is high.

A comparison of the precision of the CAV bound with the precisions of the other estimators studied presents some difficulty because the concept of a standard error is not directly applicable to the CAV bound. A reasonable method of comparing precision may be in terms of how close the lower confidence limit for the population total audit value tends to be to the true total audit value. Table 9.3 has presented these results for the CAV bound, and Table 9.4, opposite, presents comparable data for the stratified mean-per-unit estimator based on 20 strata. This latter table shows for each study population the approximate mean lower confidence limit for the population total audit value by the stratified mean-per-unit estimator based on 20 strata and a nominal 95% confidence coefficient, expressed as a percentage of the population total audit value.[6] A comparison between Tables 9.3 and 9.4 indicates that the stratified mean-per-unit estimator is more precise than the CAV bound for all study populations, particularly so for the high error rate study populations. For both estimators, the relative precision worsens as the error rate increases. This effect is somewhat more pronounced for the CAV bound than for the mean-per-unit estimator.

Since earlier comparisons of precision were made in terms of relative standard errors, it may be useful to note the implications of differences in the percentages of Tables 9.3 and 9.4. Consider the .5% error rate study population for population 3, for sample size 200. Here, the mean lower confidence limit for the CAV bound is 98.2% of the true total audit value, while for the stratified mean-per-unit estimator this percentage is 99.1%. This difference represents the equivalent of a substantial sample size, since even for sample size 100 the stratified mean-per-unit estimator leads to a closer lower confidence limit (98.6%) than does the CAV bound for sample size 200.

In making the above precision comparisons for population 4, it should be kept in mind that the proportion of correct one-sided lower confidence intervals for the mean-per-unit estimator tends to be somewhat less than the nominal confidence coefficient, while the opposite is the case for the CAV bound.

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6. The computer runs were not designed to yield explicitly the lower confidence limits for the stratified mean-per-unit estimator. Therefore, the mean lower confidence limit with a nominal 95% confidence coefficient is approximated as follows:

$$\overline{\hat{X}} - 1.645\, S(\hat{X})$$

where $\overline{\hat{X}}$—defined in formula (1.1)—is the mean of the 600 sample estimates $\hat{X}$, and $S(\hat{X})$—defined in formula (1.2)—is the standard deviation of the 600 $\hat{X}$ values. This procedure may yield reasonably good results even though there are correlations between $\hat{X}$ and $s(\hat{X})$ because the distributions of the standardized statistic Z have means close to 0 and standard deviations close to 1 in almost all cases (see Appendix Tables A-83 through A-86, pp. 194 and 195).

Table 9.4
Approximate Mean Lower Confidence Limit for Population Total Audit Value as Percent of Total Audit Value, Stratified Mean-Per-Unit Estimator With 20 Strata and Nominal 95% Confidence Coefficient

Population	Population Error Percentage .5	1	5	10	30
			$n = 100$		
3	98.6	98.5	98.5	98.4	98.2
4	98.6	98.5	97.9	96.8	93.5
			$n = 200$		
3	99.1	99.1	99.0	99.0	98.9
4	99.0	98.9	98.7	98.1	95.7

10

Dollar Unit Mean-Per-Unit Estimator

When a random sample of dollar units is selected, it is possible to use various types of estimators besides CAV bounds. This chapter presents experimental results for the mean-per-unit estimator when dollar unit sampling is employed.

Mean-Per-Unit Estimator

As before, let w_i denote the audit value of the i^{th} sample dollar unit when the error in the audit unit is assigned pro rata to all dollar units:

$$w_i = \frac{x_i}{y_i} \tag{10.1}$$

Also as before, Y denotes the population total book value, which here represents the total number of dollar units in the population. The mean-per-unit estimator of the population total audit value with dollar unit sampling is then:

$$(10.2) \qquad \hat{X} = \frac{Y}{n} \sum_{i=1}^{n} w_i = Y\bar{w}$$

where $\bar{w}$ is the sample mean of the dollar unit audit values:

$$(10.3) \qquad \bar{w} = \frac{\sum_{i=1}^{n} w_i}{n}$$

Formula (10.2) is a direct counterpart of formula (3.1) for the mean-per-unit estimator with simple random sampling of audit units. Here the population size is the total number of dollar units, namely the population total book value Y.

When random sampling of dollar units is performed with replacement, as was done for this experimental study, the estimated variance of $\hat{X}$ is:

$$(10.4) \qquad s^2(\hat{X}) = Y^2 \frac{s^2}{n}$$

where s^2 is the sample variance of the dollar unit audit values:

$$(10.5) \qquad s^2 = \frac{\sum_{i=1}^{n} (w_i - \bar{w})^2}{n - 1}$$

The only difference between formula (10.4) and its counterpart formula (3.3) for random sampling of audit units is that the finite population correction $(1 - n/N)$ is dropped here because sampling is with replacement.

The mean-per-unit estimator with dollar unit sampling has similar properties as the ratio and difference estimators with audit unit sampling. If no errors are found in the sample, all w_i equal 1 and the estimate $\hat{X}$ equals Y, the population total book value. Also, the estimated variance $s^2(\hat{X})$ equals zero when no errors are found in the sample.

The mean-per-unit estimator with dollar unit sampling with replacement may be viewed as a PPS (probability proportional to size) estimator. The reason is that dollar unit sampling with replacement, when errors are prorated to the dollar units, is equivalent to sampling of audit units with replacement, where probabilities of selection are proportional to the audit unit's book value. The mean-per-unit estimator is then equivalent to an ordinary unbiased PPS estimator where the audit values x_i are weighted inversely proportional to the probabilities of selection.[1]

Experimental Findings

Appendix Tables A-123 through A-130 (pp. 229–236) contain the basic experimental results for the mean-per-unit estimator with dollar unit sampling. Statistical theory shows that the mean-per-unit estimator in formula (10.2) is unbiased. The experimental results are in accord with this. In all cases the mean of the 600 sample estimates $\hat{X}$ is relatively very close to the population total audit value.

Precision of Mean-Per-Unit Estimator. Table 10.1, below, presents for selected study populations the standard error of the mean-per-unit estimator as a percent of the true total audit value, for sample size 100.[2] The following several comparisons with earlier results are of interest:

1. The relative standard error of the mean-per-unit estimator with dollar unit sampling tends to increase with higher error rates. The same is the case for the stratified mean-per-unit, ratio, and difference estimators

Table 10.1
Relative Standard Error of Mean-Per-Unit
Estimator, $n = 100$
(in percent)

	Population Error Percentage				
Population	.5	1	5	10	30*
1	.2	.2	–	.6	1.0
2	–	–	1.0	1.5	–
3	.1	–	.3	.5	.9
4	.5	–	1.0	1.8	3.9

*For population 2, this error percentage is 70.

1. See, for example, William G. Cochran, *Sampling Techniques,* 2nd ed. (New York: John Wiley & Sons, Inc., 1963), pp. 251–255.
2. The reason why this estimator was not investigated for all study populations is that experimental runs for dollar unit sampling are more costly than runs for sampling of audit units. For populations 3 and 4, more study populations were investigated because the CAV bound was also of interest.

and for the ratio and difference estimators with unstratified random sampling of audit units.

2. The dollar unit mean-per-unit estimator is more precise than the stratified mean-per-unit estimator (based on 20 strata) for populations 1 and 3, for all error rates investigated. It is also more precise than the stratified mean-per-unit estimator for population 4 when the error rate is low (no information is available for population 2); but for higher error rates with populations 2 and 4, the two estimators have about equal precision. Thus, dollar unit sampling achieves for the four populations all the precision benefits which stratification of audit units supplies, or even more, without requiring any stratification. Dollar unit sampling does require, however, cumulation of the population book values so that the sample dollar units can be identified.

3. The mean-per-unit estimator with dollar unit sampling tends to have about the same relative standard error as the difference and ratio estimators with unstratified random sampling of audit units for populations 1 and 2. It tends to have smaller relative standard errors for population 4 and somewhat larger relative standard errors for population 3.

The results for sample size 200 are not discussed in detail, but they correspond to those for sample size 100.

Percentage of Samples With Errors. Table 10.2, opposite, presents, for selected study populations, the percentage of samples containing at least one error with dollar unit sampling. The picture shown by this table is a familiar one—namely, a high proportion of samples from the very low error rate study populations contain no errors, thus leading to estimated standard errors of zero.[3] When the error rate reaches 5% or more, almost all samples contain at least one error.

These results did not require an empirical sampling study, to be sure, since probability theory can provide the exact probability that a sample

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3. A comparison of the percentages in Table 10.2 with the results in Table 4.4 for simple random sampling of audit units shows some differences, particularly for the .5% error rate study population for population 1. Such differences are entirely possible because the population error rate for dollar units need not be exactly the same as the error rate for audit units. Thus, the use of .5%, 1%, etc. error rate descriptions for the study populations in conjunction with dollar unit sampling is designed to provide continuity with earlier descriptions where these do denote the error rate for audit units. In the context of dollar unit sampling, however, these error rate descriptions do not show the actual error rates for dollar units.

Table 10.2
Percent of 600 Dollar Unit Samples
Containing One or More Errors

Population	Population Error Percentage .5	1	5	10	30*
			$n = 100$		
1	20.2	60.8	–	100.0	100.0
2	–	–	99.2	100.0	–
3	37.8	–	99.3	100.0	100.0
4	47.7	–	99.2	100.0	100.0
			$n = 200$		
1	39.0	87.5	–	100.0	100.0
2	–	–	100.0	100.0	–
3	68.8	–	100.0	100.0	100.0
4	71.8	–	100.0	100.0	100.0

*For population 2, this error percentage is 70.

contains one or more dollar units in error. The presentation of the empirical results has the advantage of displaying the findings in a readily understandable context.

Reliability of Nominal Confidence Coefficient. Tables 10.3 and 10.4 (pp. 122 and 123) show the actual percentages of correct confidence intervals based on the dollar unit mean-per-unit estimator for selected study populations, for sample sizes 100 and 200 respectively. Results for two confidence intervals are presented in these tables—for the two-sided, nominal 95.4% confidence interval for the population total audit value and for the one-sided lower, nominal 93.3% confidence interval. As before, a sample for which the estimated standard error is zero is considered to lead to an "incorrect" interval.

Several interesting findings emerge from Tables 10.3 and 10.4:

1. When the error rate is very low (.5%), the mean-per-unit estimator with dollar unit sampling leads to large proportions of incorrect two-sided confidence intervals, in part because many samples contain no errors.

2. When the error rate is high (10% or more), the proportions of correct two-sided confidence intervals for sample size 100 are within 5.1 percent points of the nominal 95.4% confidence level for populations 1 and 2 and, when the error rate is 30%, for population 4. On the other

Table 10.3
Actual Percent of Correct† Confidence
Intervals With Mean-Per-Unit Estimator,
$n = 100$

Population	Population Error Percentage				
	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	17.5	49.7	–	90.3	92.2
2	–	–	80.2	94.5	–
3	5.2	–	31.5	44.8	77.0
4	30.7	–	69.7	86.5	94.5
	One-sided Lower, Nominal 93.3%				
1	20.2	60.3	–	98.7	98.2
2	–	–	98.7	98.7	–
3	5.2	–	31.5	44.2	76.8
4	30.7	–	69.7	83.0	89.7

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

hand, the actual proportions of correct intervals are substantially below the nominal level for all error rate study populations for population 3. The picture is similar for sample size 200. The main distinction for sample size 200 is that the actual proportions of correct intervals for populations 2 and 4 are within 5.6 percent points of the nominal level when the error rate is already 5%.

3. The reliability of the nominal confidence coefficient for the one-sided lower confidence interval is poor when the error rate is very low (.5%). On the other hand, when the error rate is 5% or higher, the actual proportions of correct intervals are within 5.4 percent points of the nominal level for populations 1 and 2 when the sample size is 100, and within 4.7 percent points when the sample size is 200. Similar to the two-sided interval, the actual proportions of correct one-sided intervals tend to be far below the nominal 93.3% level for all error rate study populations for population 3. For population 4, the actual proportions of correct one-sided intervals are within 5.8 percent points of the nominal level for the 30% error rate study population when the sample size is 100 and for the study populations with error rates of 5% or more when the sample size is 200.

4. The major difference between the mean-per-unit estimator with dollar unit sampling and the difference and ratio estimators with random sampling of audit units, in terms of the reliability of the nominal confidence coefficient, is that the nominal confidence coefficient is more reliable for the dollar unit mean-per-unit estimator for population 4, but it is less reliable for population 3 (Tables 4.5 through 4.8, pp. 48–51).

5. The reliability of the nominal confidence coefficient for the mean-per-unit estimator with dollar unit sampling does not compare favorably with that of the mean-per-unit estimator with stratified sampling of audit units (based on 20 strata). In no case is the reliability substantially better for the dollar unit estimator and it is often much worse (Tables 7.7 and 7.8, pp. 89 and 90).

The two-sided, nominal 98.8% confidence interval and the one-sided lower, nominal 97.7% confidence interval are not discussed in detail here. Their behavior in terms of the reliability of the nominal confidence coefficient parallels that of the two confidence intervals considered above.

Table 10.4
Actual Percent of Correct† Confidence Intervals With Mean-Per-Unit Estimator, $n = 200$

	Population Error Percentage				
Population	.5	1	5	10	30*
	Two-sided, Nominal 95.4%				
1	27.5	71.7	–	92.7	93.0
2	–	–	91.5	97.3	–
3	10.7	–	47.0	62.7	88.2
4	46.0	–	89.8	94.3	95.2
	One-sided Lower, Nominal 93.3%				
1	39.0	87.0	–	97.7	97.8
2	–	–	98.0	96.7	–
3	10.7	–	46.7	62.0	85.3
4	45.7	–	88.0	87.5	92.5

† If estimated standard error is zero, interval is considered incorrect.
* For population 2, this error percentage is 70.

Part V

Conclusion

11

Summary and Implications

Summary of Major Results

Summary Chart. Figure 11.1 (p. 129) summarizes the major results of this empirical study for each of the statistical procedures studied, other than the CAV bound, for four error rates, as follows:

Low error rate—1%
Moderate error rate—5%
High error rate—10%
Very high error rate—30% (70% for population 2)

Information on two key aspects of each statistical estimator are presented in Figure 11.1:

- Precision of estimator
- Reliability of nominal confidence coefficient

Data on precision are provided in the form of the relative standard error (standard error of estimator of population total audit value as a percent of true population total audit value). All relative standard errors in Figure 11.1 are based on sample size 100. Relative standard errors for sample size 200 generally are smaller, but their comparative relations for different estimators, populations, or error rates tend to be similar to the results for sample size 100.

Data on the reliability of the nominal confidence coefficient are shown in Figure 11.1 for both sample sizes 100 and 200, so that improvements in the reliability with increasing sample size are discernible. For summary purposes, reliability of the nominal confidence coefficient is classified as follows:[1]

Degree of Reliability	Difference Between Actual Proportion of Correct Intervals and Nominal Confidence Level
High	within ± 2.5 percent points
Moderate	between ± 2.5 and ± 5.0 percent points
Fair	between ± 5.0 and ± 10.0 percent points
Low	greater than ± 10.0 percent points

The results on reliability of the nominal confidence coefficient presented in Figure 11.1 are for the two-sided, nominal 95.4% confidence interval.[2]

All results in Figure 11.1 for stratified sampling of audit units are based on 15 strata.

Following in the remainder of this section are the key results pertaining to each of the statistical procedures studied.

Results for Simple Random Sampling of Audit Units.

1. *Mean-per-unit estimator:*
   a. The mean-per-unit estimator is highly imprecise for all populations and error rates studied, compared to other estimators.
   b. The reliability of the nominal confidence coefficient is moderate to high when the moderately skewed populations 2 and 4 are sampled and the sample size is 100. For sample size 200, the reliability becomes consistently high for these two populations.
   c. The reliability of the nominal confidence coefficient is low when the more highly skewed populations 1 and 3 are sampled and the sample size is 100. The reliability becomes fair for these populations when the sample size is increased to 200.

2. *Difference and ratio estimators:*
   a. For the populations and sample sizes considered in this study, the bias of the ratio estimator is negligible.

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1. Since the actual proportions of correct intervals are presented in the text tables and in the Appendix tables, a summary table based on differently defined degrees of reliability can be readily constructed, if desired.
2. The maximum positive difference between the actual proportion of correct intervals and the nominal level for this case can be only 4.6 percent points. For ease of comprehension, the definitions of the reliability classes do not explicitly show this restriction.

Figure 11.1 Summary of Key Results

	POPULATION 1				POPULATION 2				POPULATION 3				POPULATION 4			
SKEWNESS OF BOOK VALUES	VERY HIGH				MODERATE				HIGH				MODERATE			
ERROR DIRECTION	OVER AND UNDER				OVER AND UNDER				OVER ONLY				OVER ONLY			
ERROR SIZE	SMALL				LARGE				MODERATE				LARGE			
ERROR RATE	LOW	MOD.	HIGH	VERY HIGH	LOW	MOD.	HIGH	VERY HIGH	LOW	MOD.	HIGH	VERY HIGH	LOW	MOD.	HIGH	VERY HIGH
UNSTRATIFIED																
Mean-Per-Unit	24.0	N/A	24.0	23.9	18.2	18.3	18.2	18.3	35.7	35.7	35.8	36.1	20.2	20.4	20.7	21.4
Difference	.2	.4	.6	.9	.4	1.0	1.3	3.0	.1	.1	.2	.4	1.1	1.6	3.5	9.2
Ratio	.2	.4	.6	.9	.4	1.0	1.3	3.1	.1	.2	.3	.7	1.1	1.6	3.5	8.4
MPU-Diff.	2.4	N/A	2.5	2.5	1.9	2.1	2.2	3.6	3.6	3.6	3.6	3.6	2.3	2.5	3.8	8.5
MPU-Ratio	2.4	N/A	2.4	2.5	1.9	2.1	2.2	3.5	3.6	3.6	3.7	4.1	2.3	2.7	4.2	8.5
PS-MPU	22.8	N/A	22.8	26.4	8.5	8.6	N/A	N/A	27.8	27.8	N/A	N/A	11.8	12.0	N/A	N/A
STRATIFIED*																
Mean-Per-Unit	1.1	1.2	1.3	1.5	1.0	1.2	1.4	2.5	1.2	1.2	1.2	1.3	1.2	1.4	2.0	4.1
Difference	.2	.4	.7	1.1	.3	.7	1.0	2.4	.1	.2	.3	.6	.7	1.0	1.8	4.0
Ratio	.2	.4	.7	1.1	.3	.8	1.0	2.4	.1	.2	.3	.6	.7	1.0	1.8	4.0
DOLLAR UNIT																
Mean-Per-Unit	.2	N/A	.6	1.0	N/A	1.0	1.5	N/A	N/A	.3	.5	.9	N/A	1.0	1.8	3.9

*For fifteen strata

LEGEND

1.
2.
3.

1. Reliability of nominal confidence level for sample size 100, two-sided, 95.4% interval
2. Reliability of nominal confidence level for sample size 200, two-sided, 95.4% interval
3. Relative standard error (in percent) for sample size 100

Reliability of Nominal Confidence Level - Actual Proportion of Correct Intervals:

- Within 2.5 % points of 95.4%
- Between 2.5 and 5.0 % points of 95.4%
- Between 5.0 and 10.0 % points of 95.4%
- More than 10.0 % points from 95.4%

b. The precisions of the difference and ratio estimators are practically the same for the populations and error patterns considered. These estimators have greatest precision, among all of the estimators studied, for two of the populations when the error rate is low or moderate. For the other two populations, the precisions of these estimators are fairly close to those of the best estimators when the error rate is low or moderate.
c. The relative standard errors for the difference and ratio estimators increase as the population contains higher error rates, often substantially.
d. The difference and ratio estimators are least precise for the two populations with the largest error amounts (2 and 4).
e. The reliability of the nominal confidence coefficient is quite similar for both the difference and ratio estimators.
f. The reliability of the nominal confidence coefficient is low for all populations studied when the error rate is low. The reliability is moderate to high for population 1 starting with a 5% error rate, and for population 2 starting with a 10% error rate, when the sample size is 100. For populations 3 and 4, the two populations for which all errors are overstatements, the reliability of the nominal confidence coefficient remains low with sample size 100 for all error rates, except for the highest error rate for population 3.
g. For sample size 200, the reliability of the nominal confidence coefficient tends not to remain at a low level as long, when the error rate increases, as for the smaller sample size 100.

3. *Combined mean-per-unit and auxiliary information estimators:*
   a. The combined mean-per-unit and ratio estimator (mpu-ratio estimator) has negligible bias for the populations and sample sizes considered.
   b. The combined mean-per-unit and difference estimator (mpu-difference estimator) and the mpu-ratio estimator have practically the same precisions for the populations under study. These precisions are far better than those of the mean-per-unit estimator with simple random sampling of audit units, but they are not as good as those of some other estimators.
   c. The relative standard errors of the combined estimators tend to increase as the error rate increases, but not as much relatively as those for the difference and ratio estimators.
   d. The reliability of the nominal confidence coefficient tends to be very similar for the mpu-difference and mpu-ratio estimators for the populations investigated.
   e. For the more moderately skewed populations 2 and 4, the reliability of the nominal confidence coefficient is high when the sample size is 100, except for the highest error rate study population for

population 4 for which the reliability is low. This latter reliability becomes fair for sample size 200.

f. For the more highly skewed populations 1 and 3, the reliability of the nominal confidence coefficient is low for sample size 100 and still only fair for sample size 200, for all error rates investigated.

4. *Poststratified mean-per-unit estimator:*
   a. When 7 strata are employed, the bias of the poststratified mean-per-unit estimator (ps-mpu estimator) with sample size 100 is small for all populations studied except for population 3 where the bias amounts to about 2.5%.
   b. The precision of the ps-mpu estimator tends to be somewhat better than that of the mean-per-unit estimator, chiefly for the more moderately skewed populations 2 and 4, but is substantially worse than that of several other estimators.
   c. The relative standard error of the ps-mpu estimator tends to be stable as the error rate increases for the populations under study.
   d. The reliability of the nominal confidence coefficient is low for the more highly skewed populations (1 and 3) when the sample size is 100. The reliability is still low for population 1, and becomes only fair for population 3, when the sample size is increased to 200.
   e. For the more moderately skewed populations (2 and 4), the reliability of the nominal confidence coefficient ranges from fair to moderate for sample size 100 and improves to moderate to good for sample size 200.

Results for Stratified Random Sampling of Audit Units.

1. *Stratified mean-per-unit estimator:*
   a. The mean-per-unit estimator with stratified sampling of audit units is comparatively precise. It is not as precise as the unstratified difference and ratio estimators for populations 1 and 3, the populations for which the error amounts are smallest. But the mean-per-unit estimator is as precise or more precise for the higher error rate study populations for populations 2 and 4, the populations having the largest error amounts.
   b. As the population contains a higher error rate, the relative standard error of the stratified mean-per-unit estimator (the strata being based on book values) tends to increase, most rapidly for populations 2 and 4.
   c. Increasing the number of strata from 15 to 20 leads to a smaller relative standard error. When the error rate is very high, however, this effect is not substantial for the populations under study.
   d. Planning of sample size based on book values tends to be less

effective with 20 strata than with 15 strata when errors are actually present in the population because the achieved standard error will tend to be further from the desired standard error.

e. The reliability of the nominal confidence coefficient is high for sample size 100 in all but one instance studied, where the reliability is moderate. When the sample size is increased to 200, the reliability is high in all instances.

f. There does not appear to be any pronounced tendency for the reliability of the nominal confidence coefficient to differ systematically for 15 and 20 strata.

2. *Stratified difference and ratio estimators:*

   a. For the populations, sample sizes, and numbers of strata considered, the bias of the stratified ratio estimator is negligibly small.

   b. The relative standard errors of the stratified difference and ratio estimators are practically the same for the populations studied, and increasing the number of strata from 15 to 20 has little effect on them.

   c. The stratified difference and ratio estimators are comparatively precise. The precision of these estimators tends to be better than that of the stratified mean-per-unit estimator (especially for populations 1 and 3).

   d. The reliability of the nominal confidence coefficient is quite similar for the stratified difference and ratio estimators, for the populations and sample sizes studied.

   e. The reliability of the nominal confidence coefficient is low for all populations when the error rate is low, whether the sample size is 100 or 200. The reliability is moderate to high with sample size 100 for an error rate of 5% or more for population 1, and is high with an error rate of 10% or more for population 2. Increasing the sample size to 200 leads to moderate to high reliability for both of these populations starting with a 5% error rate.

   f. For populations 3 and 4 (the two populations for which all errors are overstatements), a high reliability of the nominal confidence coefficient is only achieved in one instance for sample size 100—for a 30% error rate for population 4; otherwise, the reliability is low to fair for this sample size when the error rate is not low. Increasing the sample size to 200 brings a moderate to high reliability of the nominal confidence coefficient for a 30% error rate for population 3 and for a 10% error rate for population 4.

Results for Dollar Unit Sampling.

1. *Dollar unit mean-per-unit estimator:*

   a. The dollar unit mean-per-unit estimator is as precise or more precise than the stratified mean-per-unit estimator, for all populations studied.

b. The precision of the dollar unit mean-per-unit estimator tends to be about the same as that of the unstratified difference and ratio estimators for populations 1 and 2, better for population 4, and somewhat worse for population 3.
c. The relative standard error of the dollar unit mean-per-unit estimator tends to increase with higher error rates, for all populations considered.
d. The reliability of the nominal confidence coefficient is low for all populations studied when the error rate is low.[3] It remains low for population 3 for all error rates with sample size 100 and until the highest error rate with sample size 200. For the other three populations, the reliability of the nominal confidence coefficient becomes fair for populations 1 and 4 and good for population 2 at the 10% error rate, when the sample size is 100. Increasing the sample size to 200 improves the reliability of the nominal confidence coefficient for the dollar unit mean-per-unit estimator for these three populations. For example, when the sample size is 200, the reliability is moderate for population 2 and already fair for population 4 at the 5% error rate.

Results for One-Sided Confidence Interval. Figure 11.1 contains summary information on the reliability of the nominal confidence coefficient for the two-sided, nominal 95.4% confidence interval. The reliability of the nominal confidence coefficient for the one-sided lower, nominal 93.3% confidence interval often is comparable, although it is sometimes better and occasionally worse than the reliability of the nominal confidence coefficient for the two-sided, nominal 95.4% confidence interval. The reliability of the nominal confidence coefficient for the one-sided interval tends to be better for the mean-per-unit estimator, the mpu-difference estimator, the mpu-ratio estimator, and the ps-mpu estimator, all with simple random sampling of audit units.

CAV Procedure. The reliability of the nominal confidence coefficient for the CAV bound is moderate, by the earlier definition, since the actual proportions of correct intervals tend to be about 5 percent points greater than the nominal 95% confidence level, for all populations studied. Thus, the nominal confidence coefficient understates the actual assurance level with the CAV procedure, in all cases studied.

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3. This statement is based in part on the results for the .5% error rate for populations 3 and 4, which are not included in Figure 11.1.

The precision of the CAV bound is not as high as that of some other estimators, even when account is taken of the fact that the nominal confidence coefficient for the CAV bound understates the actual level of assurance. For example, the mean CAV bound for the .5% error rate study population for population 3, with sample size 100 and a nominal 95% confidence coefficient, is $451,077 when the actual amount of overstatement is $2,536. On the other hand, the stratified mean-per-unit estimator, based on 20 strata and a sample size of 100 with a nominal confidence level of 99%, would lead to an average bound of only approximately $281,400. In fact, this average bound for sample size 100 is almost as low as the average bound of $254,557 for the CAV procedure based on sample size 200 and a nominal 95% confidence level.

Implications for the Auditor

Since only four accounting populations were studied in this investigation, conclusions drawn must be largely tentative. Nevertheless, the auditor using statistical sampling should be concerned with some of the important implications which are suggested by the experimental results about the use of confidence intervals based on the normal distribution, the effect of increases in the standard error with increasing error levels, the use of fallback estimators, and other sensitive factors.

Use of Confidence Intervals Based on Normal Distribution. As explained in chapters one and two, confidence intervals for variables estimation are generally constructed in audit practice by relying on approximate normality for reasonably large sample sizes. This study has shown that a sample size of 100 is not always large enough for the nominal confidence coefficient of confidence intervals based on the normal distribution to be reliable. Indeed, there were several instances in this study where a sample size of 200 apparently is not even close to being large enough. Thus, nominal confidence coefficients for confidence intervals based on the normal distribution and on sample sizes the same as those considered in this study indicate the actual assurance level quite closely under the best circumstances, but under the worst circumstances are positively misleading.

When the auditor uses an interval estimate as an indication of the magnitude of the total audit value or of the total error amount, a modest discrepancy between the nominal confidence level and the actual assurance level may not be serious because the nominal confidence level is basically utilized as an indication of order of magnitude. (Similarly, materiality is an order-of-magnitude amount and not a precise number.) However, since even under favorable conditions there is a possibility that the nominal confidence coefficient based on the normal distribution differs somewhat from the actual confidence level, it would appear to be important that the auditor obtain corroborative evidence when the confidence interval leads to a borderline situation as to whether or not materiality has been exceeded. It may be noted that this judgmental assessment of borderline situations does not fit the typical framework of testing among two alternative hypotheses. The requirement there to use the confidence interval with a decision rule which leads either to accept (materiality not exceeded) or to reject (materiality exceeded) provides no judgmental leeway for the uncertainty about the actual risks of an incorrect decision resulting from the discrepancy between the nominal and actual confidence levels.

A related implication for the auditor is that approaches requiring a precise confidence level as an input to the evaluation of an overall risk must be used with care when the nominal confidence coefficient is not highly reliable. For example, the auditor must use the formula in paragraph .35 of Section 320B in *Statement on Auditing Standards No. 1* with care.[4] If the indicated confidence level for the substantive test is not matched by the actual confidence level for the sampling procedure employed, the actual combined reliability level may be less than that desired.

Variation of Standard Error. The standard statistical procedures for estimation and testing hypotheses assume that the standard error of the estimator is constant regardless of the errors present in the population. In this study, it was found, however, that the standard errors for many estimators increase with increases in the error rate, for the populations and error patterns considered. If this is generally true, auditors will need to recognize this whether the sample results are used in estimation or testing hypotheses contexts.[5]

1. When the sample results are used in the context of estimation and the sample size is determined from the variability of the book values, the actual precision of the intended estimator will be worse than planned when errors exist in the population and the standard error increases with the error rate. (Of course, if the presence of errors permits use of another estimator, its precision may be better than that planned.)
2. When the sample results are used in the context of testing hypotheses by means of a decision rule involving the confidence interval, an increase in the standard error with an increasing error rate will affect the

4. Committee on Auditing Procedure, *Statement on Auditing Standards No. 1*, (New York: American Institute of Certified Public Accountants, Inc., 1973).
5. The standard error need not necessarily increase with an increasing error rate. In this study, the same error pattern was used to generate different error rate study populations. In fact, the error pattern may vary with the error rate level.

risks of making incorrect decisions. For example, suppose that the decision rule is based on a one-sided upper confidence limit for the total error amount and states that the population is to be rejected if this upper limit exceeds a specified amount. With this procedure, an increase in the standard error with a rising error rate will not affect the risk that the population is accepted when the true total error amount is equal to the specified value. But it will affect the risk that the population is rejected when the true total error amount is less than the specified amount.[6]

Fallback Estimators. Auditors often plan the sample selection procedure and method of estimation based on certain expectations about the population and error characteristics. If these expectations turn out to be far off the mark once the sample has been selected and the planned estimation procedure consequently is no longer effective, the auditor may wish to utilize another estimator based on the actual sample selection method employed. Two implications from this study about the use of fallback estimators are the following:

1. For the more moderately skewed populations 2 and 4, a combined mean-per-unit and auxiliary information estimator tends to be more effective as a fallback than the poststratified mean-per-unit estimator, when a simple random sample of audit units has been selected and the

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6. To illustrate this, suppose that the sample size is planned to yield a standard error of $80,000 based upon book values (that is, no errors) and that the standard error increases as follows:

Error Amount	Standard Error
100,000	120,000
300,000	160,000

Suppose further that $300,000 is the specified (material) amount, that a 95% confidence coefficient is to be used and that the sample size is large enough so that the normal distribution is appropriate for evaluating the sampling risks. The fact that the standard error is $160,000 (and not $80,000 per the book values) when the error amount is $300,000 does not change the 5% risk that the population will be accepted when the error amount is equal to the specified amount. The risk that a population with no errors is rejected also remains at the planned low level, namely, at about a 2% level. However, the risk that a population with a nonmaterial $100,000 error amount will be rejected now becomes relatively high, about 49%, whereas this risk would have been only about 19% if the standard error had not increased in the presence of errors.

auxiliary information estimators do not perform well because of low or moderate error rates.

2. The mean-per-unit estimator with dollar unit sampling may not always be an effective fallback to the CAV bound when the error rate is moderately high, because of the nominal confidence coefficient's unreliability.

Choice Among Competing Estimators. In considering selecting among competing estimators, it is clear from an inspection of Figure 11.1 that the auditor will often be faced with possible trade-offs between reliability of the nominal confidence coefficient and precision. In other words, for a given set of circumstances one estimator is relatively more precise than another but its nominal confidence coefficient is less reliable.

The nature of the trade-offs for the various study populations can be perceived to some degree from Figure 11.1. For example, for study population 3 with very high error rate and sample size 100, unstratified difference estimation and stratified mean-per-unit estimation might be considered competing methods. However, although the former has a considerably lower relative standard error than the latter (.4% vs. 1.3%), the actual confidence level for unstratified difference estimation is farther away from the nominal 95.4% confidence coefficient than is the case for the stratified mean-per-unit procedure (90.8% vs. 95.8%).

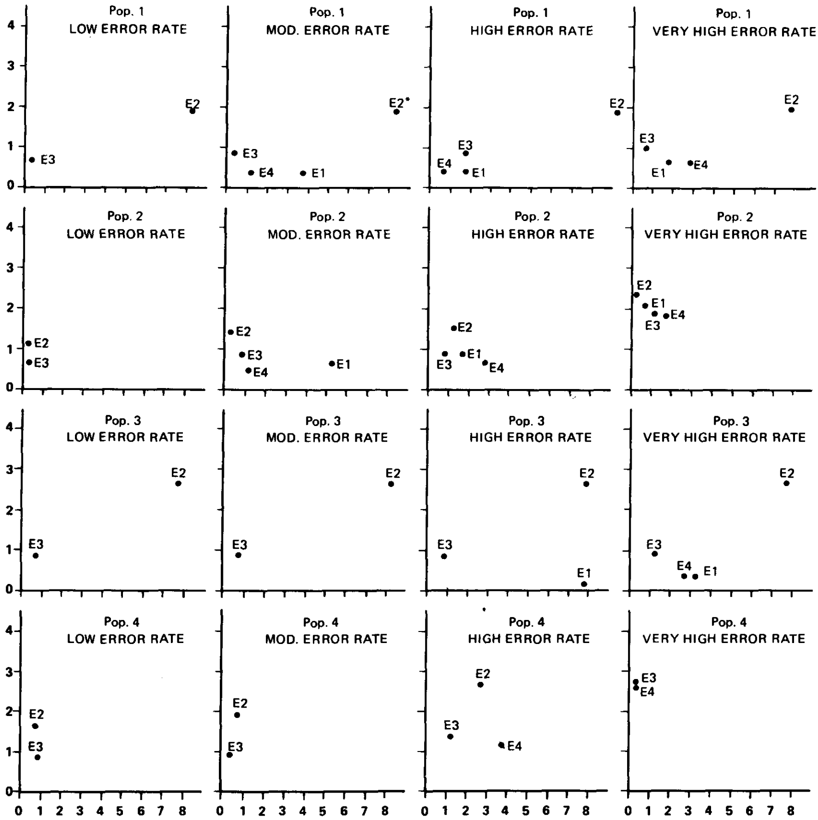
Figure 11.2 (p. 138) gives a more comprehensive view of possible trade-offs for each study population included in Figure 11.1. The vertical axis of each graph represents the relative standard error. The horizontal axis represents the absolute difference between the actual proportion of correct two-sided confidence intervals and the nominal confidence coefficient of 95.4%. The points entered on each graph are for those selected estimation methods which are within 4 percentage points on the vertical scale and within 9 percentage points on the horizontal. The selected estimation methods and their code symbols for the graphs are the following:

*E*1—Simple random sampling with difference estimation
*E*2—Simple random sampling with combined mean-per-unit and difference estimation ($w = .1$)
*E*3—Stratified mean-per-unit estimation (15 strata)
*E*4—Stratified difference estimation (15 strata)

Other estimation methods are not shown because either they would not fall within the scale of the graphs or they behave so similar to a selected method that they would provide no meaningful additional information. Finally, the sample size for all points on the graph is $n = 200$.

Inasmuch as use of the trade-offs involves subjective assessments, the auditor will need to make his choice in the context of the particular circum-

Figure 11.2
Summary of Selected Trade-Offs
(For n=200)



LEGEND:

1. VERTICAL AXIS REPRESENTS RELATIVE STANDARD ERROR AT n=200.
2. HORIZONTAL AXIS REPRESENTS ABSOLUTE DIFFERENCE BETWEEN PROPORTION OF CORRECT TWO-SIDED CONFIDENCE INTERVALS AND 95.4%.
3. SCALE NUMBERS ARE PERCENTS.
4. COORDINATE POINTS ARE SELECTED EVALUATION METHODS:
   E1 – SRS/DIFFERENCE ESTIMATION
   E2 – SRS/MPU-DIFFERENCE ESTIMATION
   E3 – STRATIFIED MPU ESTIMATION
   E4 – STRATIFIED DIFFERENCE ESTIMATION

*ESTIMATE–NO EXPERIMENTAL RUN

stances in which he must use statistical sampling.

Some Other Implications. Three other implications for the auditor are the following:

1. The behavior of the difference and ratio estimators, regarding precision and reliability of the nominal confidence coefficient, is sensitive not only to the rate of error but also to the direction and amounts of errors.
2. Unless the error rate is high, stratification for the difference and ratio estimators based on optimal procedures for the mean-per-unit estimator leads to relatively good precision for these estimators, for the populations and error patterns considered in this study. Hence, there may be little advantage in these circumstances to consider more complex stratification procedures which are optimal specifically for the difference or ratio estimators.
3. For some of the study populations, stratification based on optimal procedures for the mean-per-unit estimator does not lead to substantially more precise difference or ratio estimators than unstratified sampling of audit units. Thus, this kind of optimal stratification may not always be advantageous with the difference and ratio estimators.

Some Final Comments. It is clear from this study that no one statistical procedure is optimal under all circumstances. However, the study has also shown that, for the populations and error patterns considered, there is no situation where at least one technique is not reasonably effective. Consequently, the auditor using statistical sampling must be familiar with a variety of statistical procedures and their comparative effectiveness so that he can choose an appropriate one for any particular circumstance.

Typical of much research, this study does not provide final answers to all questions. Hence, there is a need for further investigations of some of the questions studied in this undertaking as well as of additional areas. This chapter concludes by listing some extensions and new areas for fruitful research.

Areas for Further Research

The present empirical study can be directly extended in at least two areas as follows:

1. Larger sample sizes could be studied (for example, $n = 300$ and 400) to determine when the nominal confidence coefficient becomes reliable under various conditions.
2. Smaller and larger numbers of strata than the 15 and 20 used here

could be studied to ascertain suitable stratifications for auditing uses in a variety of circumstances.

In addition, there are some important research areas which go beyond the present study as follows:

1. There is a great need for systematic information about the characteristics of accounting populations and error patterns, because the behavior of statistical estimators is sensitive to these.
2. There is a need to examine additional estimators, including:
   a. The regression estimator.
   b. Other CAV bounds, including less conservative ones and ones which handle both overstatement and understatement errors.
3. The procedure which uses a difference or a ratio estimate based on stratified sampling of audit units and an estimated standard error for the mean-per-unit estimator in constructing the confidence limits requires empirical examination.
4. The statistical procedures considered in this study and the additional ones mentioned above should be studied with other accounting populations having different characteristics and error patterns than the ones studied here so that the causes of unsatisfactory reliability of the nominal confidence coefficient can be more firmly identified.
5. There is a need to examine the statistical considerations for determining where the auditor should place the cutoff beyond which he examines audit units on a 100% basis. While audit considerations clearly must influence this decision, statistical considerations are also important for both the efficiency of the sample design and the reliability of the nominal confidence coefficient.
6. There is a need to examine the effectiveness of all statistical procedures when used in the context of testing hypotheses, for protecting the auditor against making incorrect decisions when the standard error increases with an increasing error rate.
7. Finally, it is most important that statisticians become interested in the theoretical problems raised by this and related research studies so that definitive guides can be developed about the comparative effectiveness of various statistical procedures and so that perhaps some new statistical procedures especially useful for auditing may be found.

Appendix Tables

Table A-1

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	378,817	378,709		379,360	379,918
Standard deviation	91,212	91,133		91,121	90,914
Skewness	.9	.9		.9	.9
Kurtosis	1.2	1.2		1.2	1.2
Distribution of Z					
Mean	-.7	-.7		-.7	-.7
Standard deviation	1.7	1.7		1.7	1.7
Skewness	-1.6	-1.6		-1.6	-1.6
Kurtosis	3.8	3.9		3.9	3.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89		.89	.89
Proportion of correct intervals					
Two-sided, 95.4%	81.8	81.8		81.7	81.7
Two-sided, 98.8%	86.8	86.5		86.8	87.0
One-sided, 93.3%	99.2	99.2		99.2	99.2
One-sided, 97.7%	100.0	100.0		100.0	100.0

Table A-2

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	380,470	380,465		380,546	380,911
Standard deviation	72,694	72,717		72,613	72,702
Skewness	1.3	1.3		1.3	1.3
Kurtosis	2.8	2.8		2.7	2.7
Distribution of Z					
Mean	-.5	-.5		-.5	-.5
Standard deviation	1.3	1.3		1.3	1.3
Skewness	-.9	-.9		-.9	-1.0
Kurtosis	1.0	1.0		1.0	1.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.85	.85
Proportion of correct intervals					
Two-sided, 95.4%	87.3	87.5		87.2	87.2
Two-sided, 98.8%	92.0	91.8		91.7	91.5
One-sided, 93.3%	98.2	98.3		98.3	98.2
One-sided, 97.7%	99.8	99.8		99.8	99.8

Table A-3

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 1M
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		337,180	337,041	337,404	
Standard deviation		59,993	59,783	59,924	
Skewness		.6	.6	.6	
Kurtosis		.6	.5	.5	
Distribution of Z					
Mean		-.3	-.3	-.3	
Standard deviation		1.3	1.3	1.3	
Skewness		-1.4	-1.4	-1.4	
Kurtosis		3.1	3.0	3.1	
Correlation between $\hat{X}$ and $s(\hat{X})$		.87	.87	.87	
Proportion of correct intervals					
Two-sided, 95.4%		89.8	89.8	89.2	
Two-sided, 98.8%		94.0	94.0	93.8	
One-sided, 93.3%		98.0	98.0	98.0	
One-sided, 97.7%		99.7	99.7	99.7	

Table A-4

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 1M
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,393	334,252	334,588	
Standard deviation		40,763	40,656	40,798	
Skewness		.5	.5	.5	
Kurtosis		.5	.5	.5	
Distribution of Z					
Mean		-.2	-.2	-.2	
Standard deviation		1.1	1.1	1.1	
Skewness		-1.1	-1.1	-1.1	
Kurtosis		2.1	2.2	2.0	
Correlation between $\hat{X}$ and $s(\hat{X})$		.87	.87	.87	
Proportion of correct intervals					
Two-sided, 95.4%		91.8	92.3	92.3	
Two-sided, 98.8%		96.0	95.8	95.8	
One-sided, 93.3%		97.8	97.8	97.7	
One-sided, 97.7%		99.2	99.2	99.3	

Table A-5

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,497,789	3,496,387	3,503,669	3,501,243	3,579,751
Standard deviation	634,675	635,122	637,085	636,018	651,397
Skewness	.5	.5	.5	.5	.4
Kurtosis	.8	.8	.8	.8	.6
Distribution of Z					
Mean	-.2	-.2	-.2	-.2	-.2
Standard deviation	1.1	1.1	1.1	1.1	1.1
Skewness	-.6	-.6	-.6	-.6	-.7
Kurtosis	.6	.6	.6	.5	.6
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85	.85	.85	.85
Proportion of correct intervals					
Two-sided, 95.4%	93.7	93.7	93.7	93.7	93.0
Two-sided, 98.8%	96.8	96.7	96.7	96.7	96.5
One-sided, 93.3%	96.5	96.3	96.7	96.5	96.7
One-sided, 97.7%	99.5	99.5	99.5	99.5	99.3

Table A-6

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,472,888	3,471,370	3,475,582	3,476,466	3,551,332
Standard deviation	433,614	432,947	435,124	434,274	436,644
Skewness	.4	.4	.4	.4	.5
Kurtosis	.3	.3	.3	.2	.4
Distribution of Z					
Mean	-.2	-.2	-.2	-.2	-.2
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.4	-.4	-.4	-.4	-.4
Kurtosis	.6	.6	.6	.6	.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85	.85	.85	.84
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.5	95.7	95.7	95.5
Two-sided, 98.8%	98.5	98.7	98.7	98.7	98.5
One-sided, 93.3%	94.8	94.5	95.0	94.3	95.3
One-sided, 97.7%	99.0	99.0	99.5	99.2	99.3

Table A-7

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,548,901	13,546,151	13,528,086	13,504,545	13,388,650
Standard deviation	4,878,071	4,878,281	4,878,222	4,879,034	4,880,388
Skewness	1.0	1.0	1.0	1.0	1.1
Kurtosis	2.0	2.0	2.0	2.0	2.1
Distribution of Z					
Mean	-.7	-.7	-.7	-.7	-.7
Standard deviation	1.7	1.7	1.7	1.7	1.7
Skewness	-1.9	-1.9	-1.9	-1.9	-1.9
Kurtosis	5.1	5.1	5.1	5.1	5.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.91	.91	.91	.91	.91
Proportion of correct intervals					
Two-sided, 95.4%	82.5	82.5	82.5	82.5	82.5
Two-sided, 98.8%	87.3	87.3	87.3	87.3	87.3
One-sided, 93.3%	98.0	98.0	98.0	98.0	98.0
One-sided, 97.7%	99.7	99.7	99.7	99.7	99.7

Table A-8

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,561,689	13,559,225	13,540,809	13,516,753	13,403,258
Standard deviation	3,602,508	3,602,587	3,602,930	3,604,132	3,604,753
Skewness	1.0	1.0	1.0	1.0	1.0
Kurtosis	1.7	1.7	1.7	1.7	1.7
Distribution of Z					
Mean	-.4	-.4	-.4	-.4	-.4
Standard deviation	1.4	1.4	1.4	1.4	1.4
Skewness	-1.2	-1.2	-1.2	-1.2	-1.2
Kurtosis	2.9	2.9	2.9	2.9	2.9
Correlation between $\hat{X}$ and $s(\hat{X})$	.90	.90	.90	.90	.90
Proportion of correct intervals					
Two-sided, 95.4%	87.5	87.7	87.7	87.7	87.7
Two-sided, 98.8%	92.5	92.5	92.5	92.5	92.5
One-sided, 93.3%	97.2	97.2	97.2	97.2	97.2
One-sided, 97.7%	99.2	99.2	99.2	99.2	99.2

Table A-9

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,474,863	7,464,906	7,394,091	7,234,709	6,444,499
Standard deviation	1,511,680	1,510,596	1,507,706	1,500,760	1,379,126
Skewness	.2	.2	.2	.2	.3
Kurtosis	.1	.1	.1	.2	.2
Distribution of Z					
Mean	-.2	-.2	-.2	-.2	-.2
Standard deviation	1.1	1.1	1.1	1.2	1.1
Skewness	-1.0	-1.0	-1.0	-1.1	-1.3
Kurtosis	1.9	1.9	1.8	2.6	3.8
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89	.89	.89	.89
Proportion of correct intervals					
Two-sided, 95.4%	92.7	92.7	92.3	92.8	93.2
Two-sided, 98.8%	95.3	95.5	95.3	95.2	95.5
One-sided, 93.3%	96.5	96.5	96.5	96.8	96.8
One-sided, 97.7%	99.5	99.5	99.3	99.5	99.8

Table A-10

Simple Random Sampling of Audit Units
Mean-Per-Unit Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,441,170	7,432,342	7,365,388	7,206,303	6,391,620
Standard deviation	1,054,596	1,054,838	1,057,200	1,051,413	979,255
Skewness	.4	.4	.4	.4	.5
Kurtosis	.3	.3	.3	.3	.7
Distribution of Z					
Mean	-.2	-.2	-.2	-.2	-.2
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.4	-.4	-.4	-.4	-.4
Kurtosis	1.0	.9	.9	.8	.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.88	.88	.88	.88	.89
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.2	95.2	95.0	94.7
Two-sided, 98.8%	98.8	98.8	98.7	98.3	97.7
One-sided, 93.3%	95.3	95.0	95.0	95.0	95.7
One-sided, 97.7%	99.0	99.0	99.0	99.0	99.2

Table A-11

Simple Random Sampling of Audit Units
Difference Estimator

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,169	379,249	379,140	379,686	380,174
Standard deviation	453	712	1,522	2,242	3,451
Skewness	2.0	3.1	-.3	.1	.1
Kurtosis	20.5	13.0	3.8	1.3	.4
Distribution of Z					
Number of samples with errors	228	369	594	600	600
Mean	-1.3	-2.7	.2	-.3	-.0
Standard deviation	1.9	4.3	2.5	2.4	1.0
Skewness	-.8	-1.9	18.0	-15.3	-.2
Kurtosis	.3	3.2	392.2	311.0	-.6
Correlation between $\hat{X}$ and $s(\hat{X})$	.45	.75	-.08	.21	.17
Proportion of correct intervals*					
Two-sided, 95.4%	30.5	37.3	96.8	94.0	96.3
Two-sided, 98.8%	32.3	39.0	98.2	97.5	99.5
One-sided, 93.3%	38.0	61.3	92.8	97.3	94.3
One-sided, 97.7%	38.0	61.5	97.2	99.7	99.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-12

Simple Random Sampling of Audit Units
Difference Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,180	379,289	379,096	379,466	379,863
Standard deviation	309	564	1,068	1,609	2,620
Skewness	2.3	1.9	-.4	-.2	-.0
Kurtosis	10.7	5.0	1.6	.7	.0
Distribution of Z					
Number of samples with errors	386	519	600	600	600
Mean	-1.9	-3.7	.0	-.2	-.1
Standard deviation	3.3	15.5	1.0	1.1	1.1
Skewness	-1.8	-17.6	-.0	-.3	-.2
Kurtosis	2.9	359.5	-1.1	-.6	-.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.57	.75	-.13	.09	.14
Proportion of correct intervals*					
Two-sided, 95.4%	40.0	53.0	99.2	93.5	93.7
Two-sided, 98.8%	52.5	54.8	100.0	97.8	98.3
One-sided, 93.3%	64.3	86.3	96.5	98.2	95.0
One-sided, 97.7%	64.3	86.5	99.7	99.8	99.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-13

Simple Random Sampling of Audit Units
Difference Estimator

Population 1M
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,272	334,133	334,496	
Standard deviation		653	1,682	2,091	
Skewness		2.3	-1.0	.3	
Kurtosis		12.8	5.4	1.0	
Distribution of Z					
Number of samples with errors		368	597	600	
Mean		-2.0	.1	-.3	
Standard deviation		6.3	2.5	1.1	
Skewness		-14.0	17.8	-.4	
Kurtosis		236.8	387.1	.1	
Correlation between $\hat{X}$ and $s(\hat{X})$		.52	-.21	.18	
Proportion of correct intervals*					
Two-sided, 95.4%		38.8	97.5	93.8	
Two-sided, 98.8%		48.2	98.7	97.5	
One-sided, 93.3%		61.3	92.3	98.8	
One-sided, 97.7%		61.3	98.2	100.0	

*Interval is considered incorrect if estimated standard error is zero.

Table A-14

Simple Random Sampling of Audit Units
Difference Estimator

Population 1M
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,312	334,171	334,508	
Standard deviation		490	1,071	1,572	
Skewness		2.1	-.4	-.2	
Kurtosis		7.8	1.6	.9	
Distribution of Z					
Number of samples with errors		519	600	600	
Mean		-2.7	.0	-.2	
Standard deviation		10.7	1.0	1.1	
Skewness		-17.4	-.0	-.3	
Kurtosis		354.7	-1.1	-.6	
Correlation between $\hat{X}$ and $s(\hat{X})$		.65	-.13	.07	
Proportion of correct intervals*					
Two-sided, 95.4%		53.2	99.2	93.0	
Two-sided, 98.8%		58.5	100.0	98.0	
One-sided, 93.3%		86.2	96.5	98.3	
One-sided, 97.7%		86.5	99.7	99.7	

*Interval is considered incorrect if estimated standard error is zero.

Table A-15

Simple Random Sampling of Audit Units
Difference Estimator

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,487,127	3,485,725	3,493,007	3,490,581	3,569,089
Standard deviation	5,297	13,892	35,494	45,619	108,582
Skewness	.7	-4.5	1.3	.1	.1
Kurtosis	5.0	28.0	9.3	3.7	.1
Distribution of Z					
Number of samples with errors	248	402	598	600	600
Mean	-1.6	2.2	-1.6	-.1	-.0
Standard deviation	3.3	4.0	7.1	1.2	1.0
Skewness	-1.6	2.8	-8.9	-1.2	-.3
Kurtosis	1.5	7.7	90.9	6.0	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.29	-.68	.31	-.02	.28
Proportion of correct intervals*					
Two-sided, 95.4%	31.2	41.8	82.3	97.2	95.5
Two-sided, 98.8%	31.2	49.5	85.5	99.5	99.3
One-sided, 93.3%	41.3	34.5	98.7	96.8	94.8
One-sided, 97.7%	41.3	41.8	99.7	99.7	98.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-16

Simple Random Sampling of Audit Units
Difference Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,939	3,485,421	3,489,633	3,490,517	3,565,383
Standard deviation	3,042	9,573	24,726	30,870	73,532
Skewness	.8	-3.1	.4	.0	.2
Kurtosis	4.0	13.3	3.1	1.4	.1
Distribution of Z					
Number of samples with errors	376	530	600	600	600
Mean	-3.2	3.5	-.5	-.0	-.0
Standard deviation	5.9	7.5	1.8	1.0	1.0
Skewness	-1.9	3.3	-4.0	-.1	-.1
Kurtosis	2.2	11.1	32.4	-.7	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.31	-.71	.14	-.01	.26
Proportion of correct intervals*					
Two-sided, 95.4%	44.0	54.5	90.2	97.0	96.0
Two-sided, 98.8%	44.2	61.0	94.2	99.3	99.0
One-sided, 93.3%	62.7	52.5	98.0	94.7	94.7
One-sided, 97.7%	62.7	54.5	100.0	99.2	98.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-17

Simple Random Sampling of Audit Units — Population 3
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,668,987	13,666,236	13,648,172	13,624,630	13,508,735
Standard deviation	6,970	9,327	18,925	26,720	49,219
Skewness	-4.0	-2.3	-1.3	-.9	-.4
Kurtosis	16.3	5.6	1.8	.9	.4
Distribution of Z					
Number of samples with errors	232	377	597	600	600
Mean	22.9	28.3	10.5	1.5	.3
Standard deviation	58.5	89.5	87.0	7.2	1.2
Skewness	3.2	4.2	17.8	14.9	1.4
Kurtosis	9.3	18.2	364.7	283.5	3.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.95	-.92	-.90
Proportion of correct intervals*					
Two-sided, 95.4%	23.3	36.8	73.7	80.3	90.8
Two-sided, 98.8%	25.3	37.5	77.7	83.3	94.2
One-sided, 93.3%	23.3	36.2	68.5	75.8	87.7
One-sided, 97.7%	23.3	36.8	73.7	80.3	91.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-18

Simple Random Sampling of Audit Units
Difference Estimator

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,033	13,666,569	13,648,153	13,624,097	13,510,602
Standard deviation	4,547	6,162	13,046	18,951	36,197
Skewness	-2.7	-1.7	-.9	-.7	-.5
Kurtosis	7.6	2.4	.7	.8	.3
Distribution of Z					
Number of samples with errors	377	513	600	600	600
Mean	24.6	21.3	.9	.5	.2
Standard deviation	76.2	94.7	2.5	1.6	1.1
Skewness	4.5	7.4	3.9	1.8	.4
Kurtosis	22.4	61.6	23.0	4.8	.2
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.98	-.93	-.92	-.92
Proportion of correct intervals*					
Two-sided, 95.4%	42.3	57.0	82.3	87.7	92.2
Two-sided, 98.8%	42.3	58.0	86.2	91.5	97.0
One-sided, 93.3%	42.3	53.2	77.0	82.5	88.8
One-sided, 97.7%	42.3	57.0	82.3	87.8	93.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-19

Simple Random Sampling of Audit Units
Difference Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,481,783	7,471,825	7,401,010	7,241,628	6,451,418
Standard deviation	81,253	84,025	118,625	251,699	594,818
Skewness	-5.5	-5.0	-2.3	-1.7	-.6
Kurtosis	32.5	27.3	6.8	3.2	-.2
Distribution of Z					
Number of samples with errors	251	386	596	600	600
Mean	114.5	65.8	17.5	3.6	1.5
Standard deviation	295.2	223.7	159.2	9.5	3.6
Skewness	3.2	5.0	16.2	6.5	3.2
Kurtosis	9.2	28.2	296.9	57.3	13.4
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.97	-.96	-.94
Proportion of correct intervals*					
Two-sided, 95.4%	21.2	30.0	58.2	62.0	74.8
Two-sided, 98.8%	24.0	33.0	62.2	67.2	77.8
One-sided, 93.3%	17.3	29.3	53.8	56.5	69.3
One-sided, 97.7%	21.2	30.0	58.2	62.0	74.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-20

Simple Random Sampling of Audit Units
Difference Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,473,086	7,464,259	7,397,304	7,238,220	6,423,537
Standard deviation	70,012	71,020	93,736	176,487	420,389
Skewness	-2.9	-2.7	-1.8	-1.2	-.7
Kurtosis	7.3	6.6	3.3	1.7	.8
Distribution of Z					
Number of samples with errors	384	527	600	600	600
Mean	153.6	84.4	2.3	1.8	.5
Standard deviation	438.0	390.5	5.2	4.8	1.7
Skewness	3.9	6.4	4.0	8.5	1.9
Kurtosis	16.4	44.5	23.3	120.0	4.6
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.99	-.96	-.95	-.92
Proportion of correct intervals*					
Two-sided, 95.4%	28.0	47.0	69.7	70.7	87.5
Two-sided, 98.8%	32.0	48.0	72.2	75.0	89.3
One-sided, 93.3%	26.2	42.0	66.3	65.8	82.2
One-sided, 97.7%	28.0	47.0	69.7	70.7	87.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-21

Simple Random Sampling of Audit Units
Ratio Estimator

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,169	379,242	379,164	379,725	380,294
Standard deviation	441	710	1,567	2,331	3,516
Skewness	1.9	2.8	-.0	.2	.3
Kurtosis	17.0	11.2	3.5	1.3	.5
Distribution of Z					
Number of samples with errors	228	369	594	600	600
Mean	-1.3	-2.6	.2	-.3	.0
Standard deviation	2.0	4.2	2.5	2.3	1.1
Skewness	-.8	-1.9	17.2	-14.3	-.0
Kurtosis	.3	3.1	369.3	282.6	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.41	.68	-.03	.20	.18
Proportion of correct intervals*					
Two-sided, 95.4%	29.8	36.2	94.3	91.5	92.8
Two-sided, 98.8%	32.5	40.0	97.3	96.8	97.2
One-sided, 93.3%	37.7	60.8	89.3	94.0	90.8
One-sided, 97.7%	38.0	61.3	95.8	98.7	96.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-22

Simple Random Sampling of Audit Units
Ratio Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,179	379,289	379,109	379,494	379,892
Standard deviation	314	581	1,102	1,638	2,658
Skewness	1.9	2.0	-.2	-.1	.0
Kurtosis	9.6	6.7	2.2	.7	.0
Distribution of Z					
Number of samples with errors	386	519	600	600	600
Mean	-1.9	-3.7	.0	-.2	-.1
Standard deviation	3.2	15.3	1.0	1.1	1.2
Skewness	-1.8	-17.6	-.1	-.2	-.2
Kurtosis	2.8	360.6	-.8	-.6	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.54	.71	-.09	.10	.14
Proportion of correct intervals*					
Two-sided, 95.4%	41.2	52.0	97.2	92.7	91.3
Two-sided, 98.8%	51.0	54.7	99.7	98.2	96.7
One-sided, 93.3%	64.0	85.7	95.0	94.7	92.0
One-sided, 97.7%	64.3	86.5	98.8	99.2	97.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-23

Simple Random Sampling of Audit Units
Ratio Estimator

Population 1M
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,270	334,161	334,514	
Standard deviation		660	1,623	2,064	
Skewness		2.2	-.4	.4	
Kurtosis		12.3	4.0	.9	
Distribution of Z					
Number of samples with errors		368	597	600	
Mean		-2.0	.1	-.3	
Standard deviation		6.2	2.6	1.1	
Skewness		-14.0	17.7	-.2	
Kurtosis		236.8	385.5	-.1	
Correlation between $\hat{X}$ and $s(\hat{X})$		.46	-.11	.19	
Proportion of correct intervals*					
Two-sided, 95.4%		39.8	96.2	93.7	
Two-sided, 98.8%		48.0	98.5	97.7	
One-sided, 93.3%		61.3	91.2	96.3	
One-sided, 97.7%		61.3	97.3	99.5	

*Interval is considered incorrect if estimated standard error is zero.

Table A-24

Simple Random Sampling of Audit Units
Ratio Estimator

Population 1M
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,309	334,181	334,507	
Standard deviation		485	1,078	1,561	
Skewness		2.0	-.2	-.2	
Kurtosis		8.3	1.7	.9	
Distribution of Z					
Number of samples with errors		519	600	600	
Mean		-2.7	.0	-.2	
Standard deviation		10.6	1.0	1.1	
Skewness		-17.4	-.0	-.3	
Kurtosis		355.1	-1.0	-.5	
Correlation between $\hat{X}$ and $s(\hat{X})$		.62	-.08	.08	
Proportion of correct intervals*					
Two-sided, 95.4%		53.0	99.0	93.2	
Two-sided, 98.8%		58.8	100.0	97.8	
One-sided, 93.3%		85.5	95.8	96.7	
One-sided, 97.7%		86.5	99.5	99.5	

*Interval is considered incorrect if estimated standard error is zero.

Table A-25

Simple Random Sampling of Audit Units
Ratio Estimator

Population 2
n = 100

	Population Error Percentage .5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,487,164	3,485,603	3,492,912	3,490,857	3,570,047
Standard deviation	5,644	15,198	36,342	46,556	110,877
Skewness	.8	-5.1	1.1	-.0	.2
Kurtosis	5.9	36.0	9.8	3.3	.4
Distribution of Z					
Number of samples with errors	248	402	598	600	600
Mean	-1.6	2.2	-1.6	-.1	-.0
Standard deviation	3.3	3.9	6.9	1.2	1.1
Skewness	-1.6	2.7	-8.8	-1.1	-.2
Kurtosis	1.4	7.6	90.2	5.7	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.29	-.68	.25	-.06	.24
Proportion of correct intervals*					
Two-sided, 95.4%	30.7	41.7	82.2	96.5	94.2
Two-sided, 98.8%	31.2	48.7	86.7	99.2	98.7
One-sided, 93.3%	41.2	35.5	97.7	93.8	92.0
One-sided, 97.7%	41.3	41.7	99.5	99.0	98.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-26

Simple Random Sampling of Audit Units
Ratio Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,961	3,485,503	3,489,611	3,490,670	3,567,379
Standard deviation	3,152	9,450	24,647	31,164	76,376
Skewness	1.0	-3.1	.3	-.0	.3
Kurtosis	5.4	13.4	3.4	1.3	.3
Distribution of Z					
Number of samples with errors	376	530	600	600	600
Mean	-3.1	3.4	-.5	-.0	-.0
Standard deviation	5.9	7.4	1.8	1.0	1.0
Skewness	-1.9	3.3	-4.0	-.1	.0
Kurtosis	2.2	11.1	32.1	-.6	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.32	-.69	.12	-.02	.24
Proportion of correct intervals*					
Two-sided, 95.4%	44.0	55.0	89.2	96.5	94.3
Two-sided, 98.8%	44.2	60.0	94.2	98.7	98.2
One-sided, 93.3%	62.5	51.5	97.5	94.0	92.7
One-sided, 97.7%	62.7	55.0	99.8	98.5	96.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-27

Simple Random Sampling of Audit Units
Ratio Estimator

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,668,600	13,665,375	13,644,622	13,617,361	13,484,576
Standard deviation	9,058	12,195	26,762	40,658	94,245
Skewness	-5.4	-3.4	-2.7	-2.1	-1.5
Kurtosis	35.1	14.4	13.5	7.7	3.7
Distribution of Z					
Number of samples with errors	232	377	597	600	600
Mean	21.6	26.7	9.6	1.1	-.2
Standard deviation	55.4	84.5	83.1	6.3	1.4
Skewness	3.2	4.2	18.2	14.1	-.4
Kurtosis	9.3	18.1	380.9	257.3	2.2
Correlation between $\hat{X}$ and $s(\hat{X})$	-.91	-.89	-.79	-.70	-.42
Proportion of correct intervals*					
Two-sided, 95.4%	24.2	35.2	72.8	79.5	86.2
Two-sided, 98.8%	25.8	38.0	78.2	84.0	92.3
One-sided, 93.3%	24.2	36.5	71.3	80.2	91.5
One-sided, 97.7%	24.7	37.0	75.8	84.2	95.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-28

Simple Random Sampling of Audit Units
Ratio Estimator

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,668,900	13,666,179	13,646,250	13,620,043	13,497,459
Standard deviation	4,763	6,950	16,455	25,909	63,907
Skewness	-2.8	-2.0	-1.6	-1.2	-1.2
Kurtosis	8.5	4.7	4.4	2.5	2.3
Distribution of Z					
Number of samples with errors	377	513	600	600	600
Mean	23.9	20.5	.7	.3	-.1
Standard deviation	74.1	91.5	2.4	1.5	1.2
Skewness	4.5	7.4	3.8	1.1	-.5
Kurtosis	22.2	61.3	22.5	2.8	1.0
Correlation between $\hat{X}$ and $s(\hat{X})$	-.94	-.92	-.82	-.76	-.52
Proportion of correct intervals*					
Two-sided, 95.4%	41.8	57.0	81.8	86.3	91.3
Two-sided, 98.8%	42.5	58.7	86.7	91.5	95.5
One-sided, 93.3%	42.2	55.0	79.3	84.2	92.8
One-sided, 97.7%	42.3	57.5	84.2	89.7	97.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-29

Simple Random Sampling of Audit Units
Ratio Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,482,030	7,471,919	7,399,315	7,237,750	6,458,097
Standard deviation	78,843	81,745	118,128	253,580	543,184
Skewness	-5.6	-5.0	-2.2	-1.9	-.6
Kurtosis	35.8	30.0	6.2	5.2	-.2
Distribution of Z					
Number of samples with errors	251	386	596	600	600
Mean	111.1	63.7	16.8	3.3	1.2
Standard deviation	286.5	216.4	154.6	8.8	3.1
Skewness	3.2	5.0	16.2	6.4	2.8
Kurtosis	9.2	28.2	298.9	54.0	10.5
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.98	-.95	-.91	-.84
Proportion of correct intervals*					
Two-sided, 95.4%	21.0	31.0	59.3	63.3	76.0
Two-sided, 98.8%	22.0	34.0	63.7	68.5	79.8
One-sided, 93.3%	17.7	29.8	55.5	58.7	72.5
One-sided, 97.7%	21.0	31.0	59.3	63.8	77.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-30

Simple Random Sampling of Audit Units
Ratio Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,473,163	7,464,108	7,395,459	7,234,915	6,419,780
Standard deviation	69,035	70,331	94,890	179,792	383,663
Skewness	-2.8	-2.6	-1.7	-1.3	-.4
Kurtosis	6.9	6.3	2.9	2.3	.0
Distribution of Z					
Number of samples with errors	384	527	600	600	600
Mean	151.1	83.0	2.2	1.6	.4
Standard deviation	431.3	384.3	5.0	4.5	1.6
Skewness	3.9	6.4	4.1	8.6	1.7
Kurtosis	16.5	44.5	24.0	121.8	3.7
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.98	-.94	-.92	-.84
Proportion of correct intervals*					
Two-sided, 95.4%	30.3	47.3	70.2	71.2	87.5
Two-sided, 98.8%	32.0	48.5	73.0	75.8	90.7
One-sided, 93.3%	25.7	42.5	66.2	66.3	83.8
One-sided, 97.7%	30.3	47.3	70.2	71.5	88.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-31

Simple Random Sampling of Audit Units — Population 1
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,133	379,195		379,653	380,149
Standard deviation	9,153	9,173		9,334	9,469
Skewness	.9	.8		.8	.8
Kurtosis	1.5	1.1		1.0	1.0
Distribution of Z					
Mean	-.7	-.7		-.6	-.5
Standard deviation	1.7	1.7		1.6	1.4
Skewness	-1.6	-1.6		-1.6	-1.2
Kurtosis	3.9	4.0		3.8	1.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89		.89	.87
Proportion of correct intervals					
Two-sided, 95.4%	82.3	82.0		83.3	84.3
Two-sided, 98.8%	86.7	86.3		88.2	90.7
One-sided, 93.3%	99.2	99.2		99.2	99.3
One-sided, 97.7%	100.0	100.0		99.8	99.8

Table A-32

Simple Random Sampling of Audit Units — Population 1
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,309	379,407		379,574	379,967
Standard deviation	7,284	7,322		7,325	7,641
Skewness	1.3	1.3		1.2	1.0
Kurtosis	2.7	2.7		2.2	1.8
Distribution of Z					
Mean	-.5	-.5		-.5	-.4
Standard deviation	1.3	1.3		1.3	1.3
Skewness	-.9	-.9		-.9	-1.1
Kurtosis	1.0	.9		.9	1.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.84	.83
Proportion of correct intervals					
Two-sided, 95.4%	87.2	87.0		86.2	87.7
Two-sided, 98.8%	92.0	91.5		91.8	92.2
One-sided, 93.3%	98.0	98.3		98.5	98.2
One-sided, 97.7%	99.8	99.8		99.8	100.0

Table A-33

Simple Random Sampling of Audit Units — Population 1M
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,563	334,424	334,787	
Standard deviation		6,067	6,037	6,288	
Skewness		.7	.5	.6	
Kurtosis		.7	.1	.4	
Distribution of Z					
Mean		-.3	-.3	-.3	
Standard deviation		1.3	1.3	1.3	
Skewness		-1.4	-1.2	-1.2	
Kurtosis		3.2	2.0	2.1	
Correlation between $\hat{X}$ and $s(\hat{X})$		.87	.86	.86	
Proportion of correct intervals					
Two-sided, 95.4%		90.0	90.2	89.3	
Two-sided, 98.8%		94.5	94.3	94.2	
One-sided, 93.3%		98.2	97.7	98.0	
One-sided, 97.7%		99.8	99.8	100.0	

Table A-34

Simple Random Sampling of Audit Units — Population 1M
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,320	334,180	334,516	
Standard deviation		4,144	4,137	4,412	
Skewness		.5	.5	.5	
Kurtosis		.6	.6	.5	
Distribution of Z					
Mean		-.2	-.2	-.3	
Standard deviation		1.1	1.1	1.1	
Skewness		-1.0	-1.1	-.9	
Kurtosis		1.9	2.4	1.2	
Correlation between $\hat{X}$ and $s(\hat{X})$		.86	.85	.85	
Proportion of correct intervals					
Two-sided, 95.4%		92.0	92.8	91.5	
Two-sided, 98.8%		95.7	95.8	95.8	
One-sided, 93.3%		97.5	97.5	97.7	
One-sided, 97.7%		99.2	99.2	99.7	

Table A-35

Simple Random Sampling of Audit Units — Population 2
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,488,193	3,486,792	3,494,073	3,491,647	3,570,155
Standard deviation	63,596	65,188	73,888	77,864	129,541
Skewness	.5	.3	.5	.4	.2
Kurtosis	.8	.8	1.0	1.0	-.2
Distribution of Z					
Mean	-.2	-.2	-.1	-.1	-.1
Standard deviation	1.1	1.1	1.1	1.0	1.1
Skewness	-.6	-.6	-.6	-.3	-.4
Kurtosis	.6	.7	.4	.1	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.67	.60	.42	.55
Proportion of correct intervals					
Two-sided, 95.4%	93.8	94.2	94.3	94.5	93.8
Two-sided, 98.8%	96.8	96.7	97.0	97.8	98.3
One-sided, 93.3%	96.5	96.7	96.0	95.5	95.7
One-sided, 97.7%	99.5	99.5	99.5	99.2	99.2

Table A-36

Simple Random Sampling of Audit Units — Population 2
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,485,534	3,484,016	3,488,228	3,489,112	3,563,978
Standard deviation	43,361	43,550	50,530	52,791	83,677
Skewness	.4	.4	.5	.4	.2
Kurtosis	.3	.2	.4	.3	.1
Distribution of Z					
Mean	-.2	-.2	-.2	-.1	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.4	-.4	-.4	-.3	-.3
Kurtosis	.6	.7	.7	.1	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.70	.57	.46	.52
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.2	95.2	96.7	95.2
Two-sided, 98.8%	98.5	98.8	98.7	99.2	98.2
One-sided, 93.3%	94.8	95.0	95.3	96.2	97.0
One-sided, 97.7%	99.2	98.7	98.8	99.5	99.2

Table A-37

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Difference Estimator (w = .1)

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,656,978	13,654,228	13,636,163	13,612,622	13,496,726
Standard deviation	487,915	488,161	488,352	489,489	492,401
Skewness	1.0	1.0	1.0	1.0	1.1
Kurtosis	2.0	2.0	2.0	2.0	2.1
Distribution of Z					
Mean	-.7	-.7	-.7	-.6	-.6
Standard deviation	1.7	1.7	1.7	1.7	1.6
Skewness	-1.9	-1.9	-1.8	-1.8	-1.6
Kurtosis	5.0	5.0	4.5	4.3	3.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.91	.91	.91	.91	.90
Proportion of correct intervals					
Two-sided, 95.4%	82.7	82.5	83.0	82.8	82.3
Two-sided, 98.8%	87.5	87.5	87.7	87.3	87.8
One-sided, 93.3%	98.0	98.0	97.8	98.0	98.0
One-sided, 97.7%	99.7	99.7	99.7	99.7	99.5

Table A-38

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Difference Estimator (w = .1)

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,658,298	13,655,835	13,637,419	13,613,363	13,499,867
Standard deviation	360,130	360,230	360,738	362,172	363,967
Skewness	1.0	1.0	1.0	1.0	.9
Kurtosis	1.7	1.7	1.7	1.7	1.5
Distribution of Z					
Mean	-.4	-.4	-.4	-.4	-.4
Standard deviation	1.4	1.4	1.4	1.3	1.3
Skewness	-1.2	-1.2	-1.2	-1.2	-1.1
Kurtosis	2.9	2.9	2.8	2.7	2.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.90	.90	.90	.90	.90
Proportion of correct intervals					
Two-sided, 95.4%	87.5	87.7	87.3	87.5	87.8
Two-sided, 98.8%	92.3	92.3	92.8	92.5	92.8
One-sided, 93.3%	97.2	97.2	96.8	96.5	97.3
One-sided, 97.7%	99.2	99.2	99.0	99.2	99.0

Table A-39

Simple Random Sampling of Audit Units — Population 4
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,481,091	7,471,133	7,400,318	7,240,936	6,450,726
Standard deviation	167,988	168,240	183,692	275,691	549,897
Skewness	-.2	-.2	-.2	-.9	-.6
Kurtosis	.9	.9	.5	1.6	-.1
Distribution of Z					
Mean	-.1	-.1	-.1	.3	.8
Standard deviation	1.1	1.1	1.1	1.1	1.8
Skewness	-.9	-.8	-.5	.1	1.1
Kurtosis	1.6	1.5	.6	-.7	.5
Correlation between $\hat{X}$ and $s(\hat{X})$	.04	.03	-.18	-.70	-.90
Proportion of correct intervals					
Two-sided, 95.4%	93.7	94.0	94.7	94.8	78.7
Two-sided, 98.8%	96.2	96.5	98.0	98.2	82.2
One-sided, 93.3%	95.8	96.0	94.5	86.3	72.8
One-sided, 97.7%	99.3	99.3	99.0	95.3	78.7

Table A-40

Simple Random Sampling of Audit Units — Population 4
Combined Mean-Per-Unit and Difference Estimator (w = .1) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,469,895	7,461,067	7,394,113	7,235,028	6,420,345
Standard deviation	122,370	123,100	137,916	194,745	392,803
Skewness	-.0	-.0	-.2	-.7	-.5
Kurtosis	.4	.3	.3	1.3	.5
Distribution of Z					
Mean	-.1	-.1	-.0	.3	.4
Standard deviation	1.0	1.0	1.0	1.1	1.4
Skewness	-.2	-.1	.0	.4	1.4
Kurtosis	.2	.1	-.2	-.4	2.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-.20	-.21	-.36	-.73	-.89
Proportion of correct intervals					
Two-sided, 95.4%	96.0	96.0	96.0	92.8	88.2
Two-sided, 98.8%	98.5	98.5	99.2	96.8	92.2
One-sided, 93.3%	94.7	94.5	93.0	85.5	83.8
One-sided, 97.7%	98.5	98.5	98.0	93.3	88.3

Table A-41

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Difference Estimator (w = .4)

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,028	379,033		379,555	380,072
Standard deviation	36,501	36,480		36,474	36,328
Skewness	.9	.9		.9	.9
Kurtosis	1.3	1.2		1.2	1.2
Distribution of Z					
Mean	-.7	-.7		-.7	-.7
Standard deviation	1.7	1.7		1.7	1.6
Skewness	-1.6	-1.6		-1.6	-1.5
Kurtosis	3.9	3.9		3.9	3.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89		.89	.89
Proportion of correct intervals					
Two-sided, 95.4%	81.8	82.0		81.8	82.2
Two-sided, 98.8%	86.7	86.5		87.0	87.3
One-sided, 93.3%	99.2	99.2		99.0	99.5
One-sided, 97.7%	100.0	100.0		100.0	100.0

Table A-42

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Difference Estimator (w = .4)

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,696	379,760		379,898	380,282
Standard deviation	29,085	29,111		29,008	29,122
Skewness	1.3	1.3		1.3	1.3
Kurtosis	2.8	2.8		2.7	2.6
Distribution of Z					
Mean	-.5	-.5		-.5	-.5
Standard deviation	1.3	1.3		1.3	1.3
Skewness	-.9	-.9		-.9	-1.0
Kurtosis	1.1	1.0		1.0	1.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.85	.85
Proportion of correct intervals					
Two-sided, 95.4%	87.3	87.5		87.2	87.2
Two-sided, 98.8%	91.8	91.8		91.8	91.2
One-sided, 93.3%	98.2	98.3		98.3	98.2
One-sided, 97.7%	99.8	99.8		99.8	99.8

Table A-43

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,134	379,189		379,688	380,256
Standard deviation	9,129	9,165		9,258	9,311
Skewness	.9	.8		.8	.7
Kurtosis	1.3	1.1		1.0	.9
Distribution of Z					
Mean	-.7	-.7		-.6	-.5
Standard deviation	1.7	1.7		1.6	1.4
Skewness	-1.6	-1.6		-1.7	-1.3
Kurtosis	4.0	4.0		4.0	1.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89		.88	.84
Proportion of correct intervals					
Two-sided, 95.4%	82.3	82.0		83.8	85.0
Two-sided, 98.8%	86.7	86.3		88.5	91.2
One-sided, 93.3%	99.2	99.2		99.2	99.3
One-sided, 97.7%	100.0	100.0		99.8	99.8

Table A-44

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,308	379,406		379,599	379,994
Standard deviation	7,280	7,295		7,308	7,593
Skewness	1.3	1.3		1.2	1.0
Kurtosis	2.7	2.7		2.3	1.8
Distribution of Z					
Mean	-.5	-.5		-.4	-.4
Standard deviation	1.3	1.3		1.3	1.3
Skewness	-.9	-.9		-1.0	-1.1
Kurtosis	1.1	.9		1.0	1.9
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.83	.80
Proportion of correct intervals					
Two-sided, 95.4%	87.2	87.0		86.3	88.2
Two-sided, 98.8%	92.0	91.5		91.8	92.5
One-sided, 93.3%	98.0	98.3		98.5	98.2
One-sided, 97.7%	99.8	99.8		99.8	100.0

Table A-45

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 1M
n = 100

	Population Error Percentage .5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,561	334,449	334,803	
Standard deviation		6,041	6,056	6,234	
Skewness		.7	.5	.6	
Kurtosis		.6	.2	.3	
Distribution of Z					
Mean		-.3	-.3	-.3	
Standard deviation		1.3	1.3	1.3	
Skewness		-1.4	-1.2	-1.2	
Kurtosis		3.2	2.1	2.2	
Correlation between $\hat{X}$ and $s(\hat{X})$		.87	.85	.85	
Proportion of correct intervals					
Two-sided, 95.4%		90.0	90.5	89.7	
Two-sided, 98.8%		94.5	94.5	94.7	
One-sided, 93.3%		98.2	97.7	98.0	
One-sided, 97.7%		99.8	99.8	100.0	

Table A-46

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 1M
n = 200

	Population Error Percentage .5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		334,317	334,188	334,516	
Standard deviation		4,127	4,136	4,367	
Skewness		.5	.5	.5	
Kurtosis		.5	.6	.4	
Distribution of Z					
Mean		-.2	-.2	-.3	
Standard deviation		1.1	1.1	1.1	
Skewness		-1.0	-1.1	-.9	
Kurtosis		1.9	2.5	1.3	
Correlation between $\hat{X}$ and $s(\hat{X})$		.86	.84	.82	
Proportion of correct intervals					
Two-sided, 95.4%		92.0	93.0	91.5	
Two-sided, 98.8%		95.7	95.8	96.2	
One-sided, 93.3%		97.5	97.5	97.5	
One-sided, 97.7%		99.2	99.2	99.7	

Table A-47

Simple Random Sampling of Audit Units | Population 2
Combined Mean-Per-Unit and Ratio Estimator (w = .1) | n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,488,227	3,486,681	3,493,987	3,491,896	3,571,017
Standard deviation	63,519	65,855	73,517	77,432	124,728
Skewness	.5	.2	.3	.2	-.1
Kurtosis	.8	1.0	.8	.9	-.1
Distribution of Z					
Mean	-.2	-.1	-.1	-.1	-.0
Standard deviation	1.1	1.1	1.1	1.0	1.1
Skewness	-.6	-.7	-.6	-.4	-.5
Kurtosis	.6	.6	.4	.2	-.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.84	.62	.54	.30	.33
Proportion of correct intervals					
Two-sided, 95.4%	93.8	93.8	94.2	94.5	93.8
Two-sided, 98.8%	96.8	96.7	97.0	97.8	98.2
One-sided, 93.3%	96.5	96.7	96.0	95.5	94.8
One-sided, 97.7%	99.5	99.5	99.5	99.0	98.8

Table A-48

Simple Random Sampling of Audit Units | Population 2
Combined Mean-Per-Unit and Ratio Estimator (w = .1) | n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,485,554	3,484,090	3,488,208	3,489,250	3,565,774
Standard deviation	43,323	43,693	50,163	52,529	81,108
Skewness	.4	.4	.4	.2	.0
Kurtosis	.3	.2	.3	.1	-.0
Distribution of Z					
Mean	-.2	-.2	-.2	-.1	-.0
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.4	-.4	-.4	-.3	-.2
Kurtosis	.6	.7	.7	.1	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.66	.50	.35	.31
Proportion of correct intervals					
Two-sided, 95.4%	95.2	95.2	95.3	96.5	95.0
Two-sided, 98.8%	98.5	98.8	98.5	99.2	98.5
One-sided, 93.3%	94.8	95.0	95.3	95.8	95.0
One-sided, 97.7%	99.0	98.7	98.8	99.5	98.8

Table A-49

Simple Random Sampling of Audit Units Combined Mean-Per-Unit and Ratio Estimator (w = .1)					Population 3 n = 100
	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,656,630	13,653,453	13,632,969	13,606,079	13,474,984
Standard deviation	488,797	490,002	496,772	506,227	548,923
Skewness	1.0	1.0	1.0	.9	.8
Kurtosis	2.0	2.0	1.9	1.8	1.5
Distribution of Z					
Mean	-.7	-.7	-.7	-.6	-.6
Standard deviation	1.7	1.7	1.7	1.7	1.7
Skewness	-1.9	-1.9	-1.8	-1.8	-1.7
Kurtosis	5.0	5.0	4.6	4.5	4.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.91	.91	.91	.90	.88
Proportion of correct intervals					
Two-sided, 95.4%	82.7	82.5	83.0	82.8	82.5
Two-sided, 98.8%	87.5	87.5	87.7	87.3	87.8
One-sided, 93.3%	98.0	98.0	97.8	98.0	98.0
One-sided, 97.7%	99.7	99.7	99.7	99.7	99.5

Table A-50

Simple Random Sampling of Audit Units Combined Mean-Per-Unit and Ratio Estimator (w = .1)					Population 3 n = 200
	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,658,179	13,655,484	13,635,706	13,609,714	13,488,039
Standard deviation	360,664	361,510	366,413	373,493	403,129
Skewness	1.0	.9	.9	.9	.7
Kurtosis	1.7	1.7	1.7	1.6	1.2
Distribution of Z					
Mean	-.4	-.4	-.4	-.4	-.4
Standard deviation	1.4	1.4	1.4	1.3	1.3
Skewness	-1.2	-1.2	-1.2	-1.2	-1.2
Kurtosis	2.9	2.9	2.9	2.8	2.6
Correlation between $\hat{X}$ and $s(\hat{X})$	.90	.90	.90	.89	.87
Proportion of correct intervals					
Two-sided, 95.4%	87.5	87.7	87.3	87.7	87.8
Two-sided, 98.8%	92.3	92.3	92.8	92.5	92.8
One-sided, 93.3%	97.2	97.2	96.8	96.5	97.3
One-sided, 97.7%	99.2	99.2	99.0	99.2	99.0

Table A-51

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,481,313	7,471,218	7,398,792	7,237,446	6,456,737
Standard deviation	169,307	171,127	196,587	300,838	548,828
Skewness	-.3	-.3	-.3	-1.2	-.6
Kurtosis	1.4	1.4	.6	3.4	.0
Distribution of Z					
Mean	-.1	-.1	-.1	.2	.5
Standard deviation	1.1	1.1	1.1	1.1	1.5
Skewness	-.9	-.9	-.6	-.1	.8
Kurtosis	1.6	1.5	.7	-.3	.6
Correlation between $\hat{X}$ and $s(\hat{X})$	.04	.03	-.17	-.59	-.69
Proportion of correct intervals					
Two-sided, 95.4%	93.7	94.0	94.7	93.7	81.8
Two-sided, 98.8%	96.2	96.5	97.0	97.8	87.2
One-sided, 93.3%	95.8	96.2	95.0	87.8	78.5
One-sided, 97.7%	99.3	99.3	99.0	96.0	83.7

Table A-52

Simple Random Sampling of Audit Units
Combined Mean-Per-Unit and Ratio Estimator (w = .1)

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,469,963	7,460,931	7,392,452	7,232,054	6,416,964
Standard deviation	124,780	126,508	148,037	213,778	392,933
Skewness	-.1	-.1	-.3	-.8	-.3
Kurtosis	.6	.6	.4	1.5	.1
Distribution of Z					
Mean	-.1	-.1	-.0	.2	.2
Standard deviation	1.0	1.0	1.0	1.1	1.2
Skewness	-.2	-.2	-.1	.2	1.0
Kurtosis	.3	.3	.0	-.2	1.4
Correlation between $\hat{X}$ and $s(\hat{X})$	-.21	-.21	-.35	-.64	-.70
Proportion of correct intervals					
Two-sided, 95.4%	96.0	95.8	95.5	93.3	90.3
Two-sided, 98.8%	98.5	98.5	98.8	97.0	94.7
One-sided, 93.3%	94.5	94.5	92.8	87.8	86.8
One-sided, 97.7%	98.5	98.5	97.8	94.2	91.5

Table A-53

Simple Random Sampling of Audit Units | Population 1
Combined Mean-Per-Unit and Ratio Estimator (w = .4) | n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,028	379,029		379,579	380,143
Standard deviation	36,485	36,475		36,412	36,208
Skewness	.9	.9		.9	.9
Kurtosis	1.2	1.2		1.2	1.2
Distribution of Z					
Mean	-.7	-.7		-.7	-.7
Standard deviation	1.7	1.7		1.7	1.6
Skewness	-1.6	-1.6		-1.6	-1.5
Kurtosis	3.9	3.9		3.9	3.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.89	.89		.89	.89
Proportion of correct intervals					
Two-sided, 95.4%	81.8	82.0		81.8	82.2
Two-sided, 98.8%	86.7	86.5		87.0	87.3
One-sided, 93.3%	99.2	99.2		99.0	99.3
One-sided, 97.7%	100.0	100.0		100.0	100.0

Table A-54

Simple Random Sampling of Audit Units | Population 1
Combined Mean-Per-Unit and Ratio Estimator (w = .4) | n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,695	379,759		379,915	380,300
Standard deviation	29,082	29,092		28,993	29,082
Skewness	1.3	1.3		1.3	1.3
Kurtosis	2.8	2.8		2.7	2.7
Distribution of Z					
Mean	-.5	-.5		-.5	-.5
Standard deviation	1.3	1.3		1.3	1.3
Skewness	-.9	-.9		-.9	-1.0
Kurtosis	1.1	1.0		1.0	1.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.85	.85
Proportion of correct intervals					
Two-sided, 95.4%	87.3	87.5		87.2	87.2
Two-sided, 98.8%	91.8	91.8		91.8	91.3
One-sided, 93.3%	98.2	98.3		98.3	98.2
One-sided, 97.7%	99.8	99.8		99.8	99.8

Table A-55

Simple Random Sampling of Audit Units — Population 1
Post-Stratified Mean-Per-Unit Estimator (7 strata) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	380,266	380,401		380,744	386,914
Standard deviation	86,512	86,578		86,508	100,335
Skewness	2.6	2.6		2.5	2.6
Kurtosis	13.1	13.0		12.9	11.5
Distribution of Z					
Mean	-1.3	-1.3		-1.3	-1.4
Standard deviation	2.9	2.9		2.9	3.0
Skewness	-2.6	-2.6		-2.6	-2.6
Kurtosis	9.9	10.0		9.5	11.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.92	.92		.92	.93
Proportion of correct intervals					
Two-sided, 95.4%	74.8	74.8		74.7	73.2
Two-sided, 98.8%	77.8	77.8		77.8	77.2
One-sided, 93.3%	98.8	98.8		99.0	98.7
One-sided, 97.7%	99.8	99.8		99.8	99.7

Table A-56

Simple Random Sampling of Audit Units — Population 1
Post-Stratified Mean-Per-Unit Estimator (7 strata) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	380,232	380,368		380,638	380,998
Standard deviation	68,605	68,645		68,505	68,471
Skewness	2.8	2.8		2.8	2.8
Kurtosis	18.3	18.4		18.4	18.6
Distribution of Z					
Mean	-.9	-.9		-.9	-.9
Standard deviation	2.3	2.3		2.3	2.3
Skewness	-2.8	-2.8		-2.7	-2.8
Kurtosis	12.0	12.0		11.6	12.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.90	.90		.90	.90
Proportion of correct intervals					
Two-sided, 95.4%	80.5	80.8		80.8	81.5
Two-sided, 98.8%	84.2	84.2		84.2	84.5
One-sided, 93.3%	98.7	98.7		98.5	98.3
One-sided, 97.7%	100.0	100.0		100.0	100.0

Table A-57

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 1M
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		337,180	337,105	337,527	
Standard deviation		40,314	40,136	40,447	
Skewness		.6	.6	.6	
Kurtosis		.3	.3	.3	
Distribution of Z					
Mean		-.7	-.6	-.6	
Standard deviation		2.1	2.1	2.1	
Skewness		-2.6	-2.6	-2.5	
Kurtosis		10.4	10.2	9.6	
Correlation between $\hat{X}$ and $s(\hat{X})$		.82	.81	.81	
Proportion of correct intervals					
Two-sided, 95.4%		82.3	82.0	81.8	
Two-sided, 98.8%		86.8	87.0	86.3	
One-sided, 93.3%		97.3	97.3	97.3	
One-sided, 97.7%		99.5	99.2	99.5	

Table A-58

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 1M
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		335,010	334,888	335,282	
Standard deviation		28,582	28,461	28,526	
Skewness		.4	.4	.4	
Kurtosis		.1	.2	.1	
Distribution of Z					
Mean		-.4	-.4	-.4	
Standard deviation		1.7	1.7	1.7	
Skewness		-2.6	-2.5	-2.6	
Kurtosis		11.2	10.9	11.9	
Correlation between $\hat{X}$ and $s(\hat{X})$		.81	.81	.81	
Proportion of correct intervals					
Two-sided, 95.4%		87.5	87.7	87.2	
Two-sided, 98.8%		91.3	91.3	91.7	
One-sided, 93.3%		97.2	97.0	96.8	
One-sided, 97.7%		99.8	99.8	99.8	

Table A-59

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751		
Distribution of $\hat{X}$					
Mean	3,495,836	3,494,363	3,501,034		
Standard deviation	295,708	296,284	298,921		
Skewness	.4	.4	.4		
Kurtosis	.4	.4	.4		
Distribution of Z					
Mean	-.3	-.3	-.3		
Standard deviation	1.4	1.4	1.3		
Skewness	-1.4	-1.4	-1.4		
Kurtosis	3.6	3.6	3.6		
Correlation between $\hat{X}$ and $s(\hat{X})$	.65	.65	.64		
Proportion of correct intervals					
Two-sided, 95.4%	89.2	89.0	89.5		
Two-sided, 98.8%	93.3	93.3	93.7		
One-sided, 93.3%	95.5	95.5	95.3		
One-sided, 97.7%	99.0	98.8	99.0		

Table A-60

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751		
Distribution of $\hat{X}$					
Mean	3,499,340	3,498,278	3,504,615		
Standard deviation	206,618	206,292	209,537		
Skewness	.1	.1	.1		
Kurtosis	.1	.1	.0		
Distribution of Z					
Mean	-.1	-.1	-.1		
Standard deviation	1.3	1.3	1.2		
Skewness	-1.5	-1.5	-1.4		
Kurtosis	5.5	5.6	5.4		
Correlation between $\hat{X}$ and $s(\hat{X})$	.63	.63	.63		
Proportion of correct intervals					
Two-sided, 95.4%	91.3	90.8	90.7		
Two-sided, 98.8%	95.5	95.7	96.0		
One-sided, 93.3%	94.8	94.8	94.5		
One-sided, 97.7%	99.0	98.8	98.5		

Table A-61

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829		
Distribution of $\hat{X}$					
Mean	13,333,185	13,330,606	13,312,821		
Standard deviation	3,798,791	3,798,537	3,798,649		
Skewness	.9	.9	.9		
Kurtosis	.9	.9	.9		
Distribution of Z					
Mean	-1.1	-1.1	-1.1		
Standard deviation	2.6	2.6	2.6		
Skewness	-2.7	-2.6	-2.6		
Kurtosis	11.1	11.1	11.0		
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85	.85		
Proportion of correct intervals					
Two-sided, 95.4%	76.2	76.2	76.2		
Two-sided, 98.8%	80.7	80.7	80.7		
One-sided, 93.3%	98.2	98.2	98.0		
One-sided, 97.7%	99.5	99.5	99.5		

Table A-62

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (7 strata)

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829		
Distribution of $\hat{X}$					
Mean	13,756,757	13,753,933	13,735,066		
Standard deviation	2,750,015	2,749,979	2,750,054		
Skewness	.6	.6	.6		
Kurtosis	.6	.6	.6		
Distribution of Z					
Mean	-.5	-.5	-.5		
Standard deviation	1.8	1.8	1.8		
Skewness	-2.9	-2.9	-2.9		
Kurtosis	15.5	15.5	15.5		
Correlation between $\hat{X}$ and $s(\hat{X})$	.83	.83	.83		
Proportion of correct intervals					
Two-sided, 95.4%	85.8	85.8	85.8		
Two-sided, 98.8%	90.2	90.2	90.2		
One-sided, 93.3%	96.5	96.5	96.5		
One-sided, 97.7%	99.5	99.5	99.5		

Table A-63

Simple Random Sampling of Audit Units — Population 4
Post-Stratified Mean-Per-Unit Estimator (7 strata) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350		
Distribution of $\hat{X}$					
Mean	7,451,949	7,442,929	7,378,605		
Standard deviation	880,328	879,881	885,268		
Skewness	.3	.3	.3		
Kurtosis	.2	.2	.2		
Distribution of Z					
Mean	-.2	-.2	-.2		
Standard deviation	1.3	1.3	1.3		
Skewness	-1.2	-1.2	-1.2		
Kurtosis	3.7	3.7	3.5		
Correlation between $\hat{X}$ and $s(\hat{X})$	.63	.63	.61		
Proportion of correct intervals					
Two-sided, 95.4%	91.8	91.7	92.2		
Two-sided, 98.8%	95.0	95.0	95.0		
One-sided, 93.3%	96.0	96.2	96.0		
One-sided, 97.7%	98.5	98.5	98.5		

Table A-64

Simple Random Sampling of Audit Units — Population 4
Post-Stratified Mean-Per-Unit Estimator (7 strata) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350		
Distribution of $\hat{X}$					
Mean	7,496,931	7,486,943	7,417,797		
Standard deviation	609,122	609,588	613,179		
Skewness	.2	.2	.2		
Kurtosis	-.0	-.0	-.1		
Distribution of Z					
Mean	-.1	-.1	-.1		
Standard deviation	1.1	1.1	1.1		
Skewness	-.8	-.8	-.7		
Kurtosis	2.6	2.6	2.1		
Correlation between $\hat{X}$ and $s(\hat{X})$	.59	.59	.57		
Proportion of correct intervals					
Two-sided, 95.4%	93.2	93.0	92.8		
Two-sided, 98.8%	97.2	97.3	97.2		
One-sided, 93.3%	94.3	94.2	94.0		
One-sided, 97.7%	98.2	98.0	98.0		

Table A-65

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (10 strata)

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	350,563	350,668		351,009	356,894
Standard deviation	51,284	51,346		51,364	66,454
Skewness	2.6	2.6		2.5	3.8
Kurtosis	12.9	12.8		12.5	26.0
Distribution of Z					
Mean	-3.2	-3.2		-3.1	-2.9
Standard deviation	4.2	4.3		4.1	4.1
Skewness	-2.2	-2.1		-2.0	-1.7
Kurtosis	7.0	6.9		6.0	3.9
Correlation between $\hat{X}$ and $s(\hat{X})$	.85	.85		.85	.90
Proportion of correct intervals					
Two-sided, 95.4%	50.8	50.8		51.8	52.5
Two-sided, 98.8%	57.3	57.5		57.2	58.0
One-sided, 93.3%	99.3	99.3		99.3	98.5
One-sided, 97.7%	99.7	99.5		99.5	99.2

Table A-66

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (10 strata)

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	366,554	366,671		366,967	367,325
Standard deviation	49,220	49,224		49,102	49,033
Skewness	3.4	3.4		3.3	3.3
Kurtosis	17.3	17.3		17.2	17.3
Distribution of Z					
Mean	-2.3	-2.3		-2.3	-2.2
Standard deviation	3.5	3.5		3.4	3.3
Skewness	-2.3	-2.3		-2.2	-2.1
Kurtosis	8.4	8.6		7.7	7.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.88	.88		.88	.88
Proportion of correct intervals					
Two-sided, 95.4%	55.7	56.2		56.8	57.0
Two-sided, 98.8%	62.0	62.7		62.7	64.8
One-sided, 93.3%	98.8	98.7		98.7	98.5
One-sided, 97.7%	99.0	99.2		99.3	99.2

Table A-67

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (10 strata)

Population 1M
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		318,356	318,357	318,727	
Standard deviation		21,867	21,695	22,117	
Skewness		.4	.4	.5	
Kurtosis		1.9	1.9	2.1	
Distribution of Z					
Mean		-1.9	-1.9	-1.9	
Standard deviation		2.7	2.5	2.5	
Skewness		-2.2	-2.1	-2.0	
Kurtosis		7.9	7.1	6.6	
Correlation between $\hat{X}$ and $s(\hat{X})$		.62	.62	.62	
Proportion of correct intervals					
Two-sided, 95.4%		65.0	64.8	64.7	
Two-sided, 98.8%		71.0	71.3	71.0	
One-sided, 93.3%		99.2	99.5	99.2	
One-sided, 97.7%		99.7	99.8	99.7	

Table A-68

Simple Random Sampling of Audit Units
Post-Stratified Mean-Per-Unit Estimator (10 strata)

Population 1M
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value		334,303	334,162	334,618	
Distribution of $\hat{X}$					
Mean		326,363	326,275	326,649	
Standard deviation		14,046	13,988	14,147	
Skewness		-.6	-.6	-.6	
Kurtosis		.9	.9	.9	
Distribution of Z					
Mean		-1.2	-1.2	-1.2	
Standard deviation		2.3	2.2	2.2	
Skewness		-2.0	-1.9	-1.8	
Kurtosis		6.6	5.8	5.7	
Correlation between $\hat{X}$ and $s(\hat{X})$		.31	.31	.32	
Proportion of correct intervals					
Two-sided, 95.4%		73.8	73.7	73.8	
Two-sided, 98.8%		80.2	79.8	80.5	
One-sided, 93.3%		97.2	97.7	96.8	
One-sided, 97.7%		98.5	98.5	98.7	

Table A-69

Simple Random Sampling of Audit Units — Population 1
Post-Stratified Mean-Per-Unit Estimator (15 strata) — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value					379,921
Distribution of $\hat{X}$					
Mean					345,799
Standard deviation					45,049
Skewness					2.7
Kurtosis					16.5
Distribution of Z					
Mean					-3.7
Standard deviation					4.4
Skewness					-2.0
Kurtosis					5.7
Correlation between $\hat{X}$ and $s(\hat{X})$					.88
Proportion of correct intervals					
Two-sided, 95.4%					46.0
Two-sided, 98.8%					51.2
One-sided, 93.3%					100.0
One-sided, 97.7%					100.0

Table A-70

Simple Random Sampling of Audit Units — Population 1
Post-Stratified Mean-Per-Unit Estimator (15 strata) — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value					379,921
Distribution of $\hat{X}$					
Mean					359,592
Standard deviation					44,149
Skewness					5.0
Kurtosis					37.1
Distribution of Z					
Mean					-3.1
Standard deviation					4.0
Skewness					-2.4
Kurtosis					8.3
Correlation between $\hat{X}$ and $s(\hat{X})$					.92
Proportion of correct intervals					
Two-sided, 95.4%					51.5
Two-sided, 98.8%					58.3
One-sided, 93.3%					100.0
One-sided, 97.7%					100.0

Table A-71

Stratified Random Sampling of Audit Units (15 strata) — Population 1
Mean-Per-Unit Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,034	379,064	378,875	379,279	379,782
Standard deviation	4,279	4,240	4,553	4,792	5,554
Skewness	.0	-.1	.1	.2	.4
Kurtosis	.2	.1	.2	.6	.4
Distribution of Z					
Mean	-.0	-.1	-.1	-.1	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.1	-.2	-.1	-.3	-.1
Kurtosis	.2	.2	.0	.0	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.19	.14	.30	.37	.42
Proportion of correct intervals					
Two-sided, 95.4%	94.2	94.2	94.7	95.0	95.5
Two-sided, 98.8%	98.7	98.7	98.7	98.5	98.8
One-sided, 93.3%	94.7	94.8	94.7	96.0	95.0
One-sided, 97.7%	98.0	98.0	98.3	99.0	98.5

Table A-72

Stratified Random Sampling of Audit Units (15 strata) — Population 1
Mean-Per-Unit Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,159	379,247	379,034	379,500	379,975
Standard deviation	2,708	2,731	2,975	2,982	3,863
Skewness	-.0	-.0	-.0	-.0	.2
Kurtosis	-.2	-.1	-.2	-.3	-.0
Distribution of Z					
Mean	-.0	-.0	-.1	-.1	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.1	-.1	-.2	-.2	-.2
Kurtosis	-.2	-.1	-.2	-.4	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.15	.20	.33	.33	.46
Proportion of correct intervals					
Two-sided, 95.4%	96.3	95.8	95.8	97.2	96.0
Two-sided, 98.8%	99.0	98.8	98.8	99.7	98.7
One-sided, 93.3%	94.8	94.5	95.2	96.5	94.3
One-sided, 97.7%	98.3	98.2	98.8	99.2	99.0

Table A-73

Stratified Random Sampling of Audit Units (15 strata) Population 2
Mean-Per-Unit Estimator n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,485,876	3,484,419	3,490,830	3,490,486	3,563,527
Standard deviation	34,002	35,523	42,556	49,209	90,890
Skewness	.1	.1	-.3	-.1	.0
Kurtosis	-.3	-.1	.3	.0	-.3
Distribution of Z					
Mean	-.0	-.0	-.0	-.0	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	.1	.1	-.1	-.1	-.3
Kurtosis	-.2	-.1	-.2	-.2	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.11	-.03	.02	.03	.36
Proportion of correct intervals					
Two-sided, 95.4%	96.3	96.2	96.7	96.0	94.7
Two-sided, 98.8%	99.0	98.8	99.2	99.2	99.3
One-sided, 93.3%	93.5	93.7	94.2	93.8	95.0
One-sided, 97.7%	98.0	97.8	98.3	98.2	99.2

Table A-74

Stratified Random Sampling of Audit Units (15 strata) Population 2
Mean-Per-Unit Estimator n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,487,661	3,486,405	3,491,129	3,492,692	3,569,877
Standard deviation	23,657	24,701	29,787	33,146	69,148
Skewness	.0	-.0	.0	-.0	-.0
Kurtosis	.1	-.0	.1	.1	.2
Distribution of Z					
Mean	.0	.0	.0	.0	.0
Standard deviation	1.0	1.0	1.0	1.0	1.1
Skewness	-.1	-.0	.0	.0	-.4
Kurtosis	.1	-.0	-.2	-.1	.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.20	-.06	.03	.05	.41
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.8	96.2	96.3	94.3
Two-sided, 98.8%	98.2	98.2	99.0	98.8	98.2
One-sided, 93.3%	95.0	93.8	94.0	93.7	93.2
One-sided, 97.7%	97.8	97.8	98.2	98.0	98.2

Table A-75

Stratified Random Sampling of Audit Units (15 strata) — Population 3
Mean-Per-Unit Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,668,343	13,666,014	13,647,379	13,622,519	13,512,208
Standard deviation	165,588	166,086	167,349	169,630	178,022
Skewness	.1	.1	.1	.1	-.0
Kurtosis	-.2	-.2	-.2	-.1	-.1
Distribution of Z					
Mean	-.0	-.0	-.0	-.0	.0
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.0	-.0	.0	.0	.0
Kurtosis	-.2	-.2	-.2	-.1	-.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.17	.13	.09	.04	-.06
Proportion of correct intervals					
Two-sided, 95.4%	95.5	95.5	95.8	95.7	95.8
Two-sided, 98.8%	98.3	98.5	98.5	98.7	98.8
One-sided, 93.3%	93.5	93.0	93.2	93.8	93.8
One-sided, 97.7%	97.7	97.5	97.8	97.5	98.0

Table A-76

Stratified Random Sampling of Audit Units (15 strata) — Population 3
Mean-Per-Unit Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,670,243	13,667,703	13,649,604	13,622,651	13,507,894
Standard deviation	105,259	105,595	107,034	107,838	115,357
Skewness	-.0	-.0	-.1	-.1	-.0
Kurtosis	-.1	-.1	-.1	-.2	-.5
Distribution of Z					
Mean	.0	.0	.0	-.0	-.0
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.1	-.1	-.1	-.1	.0
Kurtosis	-.1	-.1	-.1	-.2	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.29	.26	.18	.11	-.08
Proportion of correct intervals					
Two-sided, 95.4%	94.7	94.7	94.8	96.2	96.7
Two-sided, 98.8%	98.7	98.7	98.7	98.7	99.3
One-sided, 93.3%	93.0	93.2	94.2	93.8	94.0
One-sided, 97.7%	98.0	97.8	97.8	98.5	98.0

Table A-77

Stratified Random Sampling of Audit Units (15 strata) — Population 4
Mean-Per-Unit Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,479,231	7,469,536	7,402,234	7,233,472	6,430,633
Standard deviation	83,038	86,496	101,548	143,807	262,401
Skewness	-.2	-.1	-.3	-.2	-.1
Kurtosis	-.0	-.1	-.2	-.1	-.4
Distribution of Z					
Mean	.1	.1	.2	.2	.1
Standard deviation	1.0	1.1	1.1	1.2	1.1
Skewness	.3	.3	.5	1.0	.6
Kurtosis	-.3	-.3	-.1	1.1	.5
Correlation between $\hat{X}$ and $s(\hat{X})$	-.49	-.53	-.67	-.83	-.89
Proportion of correct intervals					
Two-sided, 95.4%	94.7	94.7	93.3	91.0	95.0
Two-sided, 98.8%	98.8	98.5	97.3	93.5	97.8
One-sided, 93.3%	89.5	89.5	87.5	85.7	90.5
One-sided, 97.7%	95.5	95.3	94.0	91.0	96.0

Table A-78

Stratified Random Sampling of Audit Units (15 strata) — Population 4
Mean-Per-Unit Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,481,010	7,470,462	7,402,691	7,242,453	6,443,802
Standard deviation	52,352	53,828	65,792	91,746	176,396
Skewness	-.1	-.1	-.2	-.2	.1
Kurtosis	.1	.1	-.3	.1	.1
Distribution of Z					
Mean	.1	.1	.1	.2	.1
Standard deviation	1.0	1.0	1.0	1.1	1.0
Skewness	.3	.4	.3	.8	.7
Kurtosis	.0	.2	-.2	1.3	1.6
Correlation between $\hat{X}$ and $s(\hat{X})$	-.48	-.51	-.67	-.82	-.91
Proportion of correct intervals					
Two-sided, 95.4%	94.8	96.2	95.7	94.2	95.0
Two-sided, 98.8%	98.5	98.2	98.5	96.8	98.2
One-sided, 93.3%	91.7	91.7	89.5	90.3	91.0
One-sided, 97.7%	95.7	96.5	96.0	94.5	95.7

Table A-79

Stratified Random Sampling of Audit Units (20 strata)
Mean-Per-Unit Estimator

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,066	379,187	379,082	379,548	380,241
Standard deviation	3,313	3,358	4,018	4,399	5,470
Skewness	-.0	.0	.6	.6	.6
Kurtosis	.0	.2	2.0	1.4	1.6
Distribution of Z					
Mean	-.0	-.0	-.1	-.1	-.1
Standard deviation	1.0	1.0	1.1	1.1	1.0
Skewness	-.2	-.1	-.2	-.1	-.4
Kurtosis	.0	-.0	-.2	-.1	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.14	.18	.44	.46	.54
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.0	94.3	93.8	95.2
Two-sided, 98.8%	98.2	98.3	98.3	97.8	98.0
One-sided, 93.3%	93.5	93.7	92.8	94.5	95.2
One-sided, 97.7%	98.3	98.2	98.3	98.8	99.5

Table A-80

Stratified Random Sampling of Audit Units (20 strata)
Mean-Per-Unit Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,158	379,259	379,024	379,458	379,877
Standard deviation	2,095	2,130	2,409	2,683	3,419
Skewness	.0	.0	.1	.1	.5
Kurtosis	-.1	-.1	.1	.2	1.3
Distribution of Z					
Mean	-.0	-.0	-.1	-.1	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.0	-.0	-.2	-.1	-.3
Kurtosis	-.0	-.0	-.2	.0	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.15	.16	.35	.36	.52
Proportion of correct intervals					
Two-sided, 95.4%	95.3	95.8	94.7	94.2	93.8
Two-sided, 98.8%	98.3	98.3	99.0	98.3	98.0
One-sided, 93.3%	93.8	93.5	94.5	95.3	95.0
One-sided, 97.7%	97.8	98.3	98.5	98.0	98.7

Table A-81

Stratified Random Sampling of Audit Units (20 strata) | Population 2
Mean-Per-Unit Estimator | n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,052	3,485,146	3,487,726	3,487,008	3,564,510
Standard deviation	25,042	26,522	38,922	45,702	90,527
Skewness	.1	.1	-.3	-.2	.3
Kurtosis	.0	.4	.8	.4	.2
Distribution of Z					
Mean	-.0	-.0	-.1	-.1	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.0	-.0	-.0	.0	-.1
Kurtosis	-.2	-.1	-.5	-.6	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.14	.01	-.18	-.16	.24
Proportion of correct intervals					
Two-sided, 95.4%	96.0	96.8	96.5	97.3	95.8
Two-sided, 98.8%	99.7	99.7	99.7	100.0	98.7
One-sided, 93.3%	93.5	94.2	95.3	95.5	94.7
One-sided, 97.7%	98.7	98.5	98.3	99.2	98.8

Table A-82

Stratifeid Random Sampling of Audit Units (20 strata) | Population 2
Mean-Per-Unit Estimator | n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,670	3,485,011	3,489,935	3,489,339	3,561,246
Standard deviation	17,614	19,049	25,121	32,510	62,880
Skewness	.1	-.1	-.2	-.3	.3
Kurtosis	-.3	-.1	-.1	-.2	.2
Distribution of Z					
Mean	-.0	-.0	-.0	-.0	-.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	.0	.0	-.0	-.2	.0
Kurtosis	-.3	-.4	-.6	-.6	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.22	-.10	-.10	-.08	.31
Proportion of correct intervals					
Two-sided, 95.4%	96.7	96.5	96.5	95.3	96.0
Two-sided, 98.8%	99.3	99.5	99.8	99.5	99.3
One-sided, 93.3%	93.3	92.2	94.5	94.8	94.3
One-sided, 97.7%	98.7	98.2	98.2	98.5	98.2

Table A-83

Stratified Random Sampling of Audit Units (20 strata) — Population 3
Mean-Per-Unit Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,013	13,665,290	13,648,357	13,624,112	13,504,100
Standard deviation	119,911	121,684	124,889	131,139	145,198
Skewness	.2	.2	.2	.1	.2
Kurtosis	.0	.0	.0	-.1	-.0
Distribution of Z					
Mean	-.0	-.0	.0	.0	-.0
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	.2	.2	.2	.2	.3
Kurtosis	-.1	-.1	-.1	-.2	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.18	.11	-.04	-.13	-.26
Proportion of correct intervals					
Two-sided, 95.4%	96.7	96.8	96.3	96.3	94.8
Two-sided, 98.8%	98.8	98.8	99.0	98.7	98.7
One-sided, 93.3%	93.8	93.7	93.0	91.0	92.2
One-sided, 97.7%	98.2	98.3	97.7	97.3	96.5

Table A-84

Stratified Random Sampling of Audit Units (20 strata) — Population 3
Mean-Per-Unit Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,634	13,666,752	13,648,260	13,622,402	13,513,563
Standard deviation	78,667	78,261	80,006	81,953	93,373
Skewness	.0	.0	-.1	-.1	-.2
Kurtosis	-.2	-.2	-.2	.0	.1
Distribution of Z					
Mean	-.0	-.0	.0	-.0	.1
Standard deviation	1.0	1.0	1.0	1.0	1.0
Skewness	-.0	-.0	-.1	-.0	-.0
Kurtosis	-.2	-.2	-.2	-.2	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.21	.21	.06	-.05	-.29
Proportion of correct intervals					
Two-sided, 95.4%	96.0	96.3	96.3	95.8	95.7
Two-sided, 98.8%	99.0	99.2	99.3	99.3	98.8
One-sided, 93.3%	93.7	93.8	94.3	93.7	92.2
One-sided, 97.7%	97.8	98.0	98.3	98.2	97.8

Table A-85

Stratified Random Sampling of Audit Units (20 strata)
Mean-Per-Unit Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,476,352	7,466,410	7,399,074	7,233,787	6,441,614
Standard deviation	64,218	68,792	92,098	138,862	253,650
Skewness	-.4	-.5	-.6	-.6	-.2
Kurtosis	.4	.6	.3	1.1	.0
Distribution of Z					
Mean	.1	.1	.2	.3	.1
Standard deviation	1.0	1.0	1.2	1.3	1.0
Skewness	.3	.3	.5	1.2	.6
Kurtosis	-.2	-.2	.0	1.6	.7
Correlation between $\hat{X}$ and $s(\hat{X})$	-.60	-.64	-.77	-.85	-.89
Proportion of correct intervals					
Two-sided, 95.4%	95.8	95.8	91.8	89.2	94.2
Two-sided, 98.8%	98.8	98.7	96.0	92.0	97.8
One-sided, 93.3%	90.7	89.7	82.5	86.2	90.0
One-sided, 97.7%	96.5	96.5	92.2	89.8	95.2

Table A-86

Stratified Random Sampling of Audit Units (20 strata)
Mean-Per-Unit Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,479,059	7,468,885	7,403,789	7,239,992	6,446,363
Standard deviation	46,259	48,568	60,165	87,082	170,654
Skewness	-.5	-.5	-.4	-.3	-.2
Kurtosis	.4	.3	.4	-.1	-.0
Distribution of Z					
Mean	.2	.2	.2	.2	.1
Standard deviation	1.0	1.1	1.1	1.0	1.0
Skewness	.2	.3	.7	.9	.3
Kurtosis	-.3	-.3	.5	2.6	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-.61	-.64	-.75	-.86	-.91
Proportion of correct intervals					
Two-sided, 95.4%	92.3	93.7	92.5	95.8	94.8
Two-sided, 98.8%	98.5	97.8	96.0	97.7	97.8
One-sided, 93.3%	88.0	87.2	86.2	90.3	91.7
One-sided, 97.7%	93.8	94.5	93.0	95.8	95.7

Table A-87

Stratified Random Sampling of Audit Units (15 strata) — Population 1
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,167	379,197	379,007	379,411	379,915
Standard deviation	865	709	1,676	2,539	4,009
Skewness	6.8	.3	2.2	1.5	1.1
Kurtosis	83.2	19.5	12.9	5.1	2.7
Distribution of Z					
Number of samples with errors	118	378	595	600	600
Mean	-.9	-2.8	-.3	-.4	-.2
Standard deviation	1.0	22.1	1.1	1.2	1.0
Skewness	.7	-9.4	.5	-1.0	-.1
Kurtosis	-.2	87.6	-.3	6.0	-.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.63	-.05	.52	.50	.56
Proportion of correct intervals*					
Two-sided, 95.4%	18.3	55.2	96.8	92.3	95.3
Two-sided, 98.8%	19.0	56.7	98.5	97.0	99.0
One-sided, 93.3%	19.7	63.0	95.2	98.5	96.7
One-sided, 97.7%	19.7	63.0	98.0	99.7	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-88

Stratified Random Sampling of Audit Units (15 strata) — Population 1
Difference Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,159	379,248	379,034	379,500	379,975
Standard deviation	483	584	1,221	1,688	2,800
Skewness	4.2	4.1	1.4	1.0	.6
Kurtosis	36.7	26.1	3.9	2.7	1.3
Distribution of Z					
Number of samples with errors	245	549	600	600	600
Mean	-1.4	-2.6	-.4	-.3	-.2
Standard deviation	1.4	26.2	1.0	1.0	1.1
Skewness	.1	-16.3	.0	-.2	-.3
Kurtosis	.1	266.5	-1.0	-.6	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.59	.67	.58	.53	.60
Proportion of correct intervals*					
Two-sided, 95.4%	27.3	81.7	94.3	94.7	92.7
Two-sided, 98.8%	35.5	85.0	99.0	98.3	97.5
One-sided, 93.3%	40.8	91.5	98.7	98.8	96.0
One-sided, 97.7%	40.8	91.5	99.8	99.8	99.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-89

Stratified Random Sampling of Audit Units (15 strata)
Difference Estimator

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,670	3,485,213	3,491,624	3,491,280	3,564,321
Standard deviation	6,097	10,414	26,170	34,253	84,159
Skewness	2.3	-.7	-.0	.1	.0
Kurtosis	22.3	11.4	2.0	.7	-.2
Distribution of Z					
Number of samples with errors	157	347	597	600	600
Mean	-.9	1.3	-.8	-.1	-.1
Standard deviation	1.5	3.2	4.0	1.1	1.0
Skewness	-1.0	2.3	-8.8	-.2	-.3
Kurtosis	1.3	5.1	101.5	-.8	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.35	-.30	.10	.10	.36
Proportion of correct intervals*					
Two-sided, 95.4%	20.5	43.0	87.2	97.2	95.5
Two-sided, 98.8%	22.0	48.3	91.5	98.8	98.2
One-sided, 93.3%	26.2	40.3	98.5	96.2	95.7
One-sided, 97.7%	26.2	43.0	99.5	100.0	99.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-90

Stratified Random Sampling of Audit Units (15 strata) — Population 2
Difference Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,487,032	3,485,776	3,490,500	3,492,062	3,569,247
Standard deviation	4,704	7,856	18,503	24,171	65,460
Skewness	1.4	-.8	.3	.2	.1
Kurtosis	7.4	3.0	.8	.4	.2
Distribution of Z					
Number of samples with errors	298	489	600	600	600
Mean	-1.2	1.3	-.1	-.0	.0
Standard deviation	2.6	3.9	1.1	1.0	1.1
Skewness	-2.7	4.6	-.6	-.2	-.3
Kurtosis	9.0	24.3	1.1	-.8	.3
Correlation between $\hat{X}$ and $s(\hat{X})$	.38	-.25	.13	.13	.41
Proportion of correct intervals*					
Two-sided, 95.4%	42.0	65.2	96.5	98.3	93.8
Two-sided, 98.8%	42.0	67.8	97.8	99.3	98.0
One-sided, 93.3%	49.7	64.0	97.0	96.0	92.2
One-sided, 97.7%	49.7	65.2	99.7	99.3	97.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-91

Stratified Random Sampling of Audit Units (15 strata)
Difference Estimator

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,129	13,666,801	13,648,166	13,623,305	13,512,995
Standard deviation	9,965	15,415	29,704	44,161	74,778
Skewness	-5.5	-4.8	-1.6	-1.4	-.6
Kurtosis	33.2	29.0	3.2	2.3	.2
Distribution of Z					
Number of samples with errors	227	355	595	600	600
Mean	80.2	75.7	21.7	9.5	1.0
Standard deviation	77.0	126.2	60.4	22.4	6.2
Skewness	.5	2.0	12.7	3.2	10.5
Kurtosis	-.9	2.9	220.2	12.8	119.4
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.97	-.94	-.89
Proportion of correct intervals*					
Two-sided, 95.4%	8.7	14.8	48.3	67.3	86.8
Two-sided, 98.8%	8.7	14.8	49.0	69.8	90.7
One-sided, 93.3%	8.7	14.8	47.5	63.5	83.5
One-sided, 97.7%	8.7	14.8	48.3	67.3	87.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-92

Stratified Random Sampling of Audit Units (15 strata)
Difference Estimator

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,016	13,666,476	13,648,378	13,621,425	13,506,668
Standard deviation	7,171	10,730	21,396	32,202	52,734
Skewness	-4.1	-2.8	-1.1	-.8	-.2
Kurtosis	19.1	8.7	1.0	.5	-.1
Distribution of Z					
Number of samples with errors	389	525	600	600	600
Mean	146.3	104.0	11.9	3.1	.2
Standard deviation	151.1	207.1	23.8	13.0	1.2
Skewness	.5	2.5	2.3	6.0	1.6
Kurtosis	-1.0	5.1	6.3	39.0	5.3
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.99	-.95	-.93	-.90
Proportion of correct intervals*					
Two-sided, 95.4%	18.7	29.3	63.0	81.2	92.8
Two-sided, 98.8%	18.7	30.3	67.0	82.7	95.7
One-sided, 93.3%	18.7	28.0	61.5	77.2	87.7
One-sided, 97.7%	18.7	29.3	63.0	81.2	93.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-93

Stratified Random Sampling of Audit Units (15 strata)
Difference Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,874	7,469,179	7,401,877	7,233,115	6,430,277
Standard deviation	44,363	48,823	77,371	130,314	256,790
Skewness	-1.8	-1.6	-1.0	-.4	-.0
Kurtosis	2.3	2.0	.7	.1	-.4
Distribution of Z					
Number of samples with errors	261	415	597	600	600
Mean	11.8	8.1	13.7	.8	.1
Standard deviation	32.6	26.6	259.5	3.3	1.1
Skewness	6.0	8.5	24.3	5.2	.8
Kurtosis	40.2	88.5	590.1	40.4	.9
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.98	-.95	-.94	-.94
Proportion of correct intervals*					
Two-sided, 95.4%	29.5	39.7	73.0	86.2	95.2
Two-sided, 98.8%	29.5	41.7	76.2	89.7	97.3
One-sided, 93.3%	29.5	39.5	69.3	83.2	89.7
One-sided, 97.7%	29.5	39.7	73.0	86.5	95.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-94

Stratified Random Sampling of Audit Units (15 strata) | Population 4
Difference Estimator | n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,480,416	7,469,868	7,402,097	7,241,859	6,443,208
Standard deviation	30,373	33,718	52,321	86,604	170,153
Skewness	-1.2	-1.0	-.4	-.2	.2
Kurtosis	.4	.3	-.6	-.1	.2
Distribution of Z					
Number of samples with errors	399	548	600	600	600
Mean	22.7	12.1	1.2	.4	.1
Standard deviation	65.8	48.7	4.9	1.4	1.1
Skewness	6.3	10.5	10.3	2.0	1.0
Kurtosis	45.0	132.5	155.8	7.0	2.7
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.97	-.94	-.94	-.95
Proportion of correct intervals*					
Two-sided, 95.4%	43.5	60.2	81.7	91.7	95.0
Two-sided, 98.8%	43.5	61.8	85.5	94.5	97.7
One-sided, 93.3%	42.3	58.5	77.2	85.0	90.5
One-sided, 97.7%	43.5	60.2	81.7	91.8	95.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-95

Stratified Random Sampling of Audit Units (20 strata) — Population 1
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,158	379,278	379,174	379,639	380,333
Standard deviation	768	825	2,003	2,767	4,178
Skewness	7.2	6.3	2.6	2.1	1.0
Kurtosis	98.1	73.0	12.4	9.5	1.7
Distribution of Z					
Number of samples with errors	141	373	591	600	600
Mean	-1.1	-5.4	1.3	-.4	-.2
Standard deviation	1.0	36.3	19.4	1.3	1.1
Skewness	1.1	-7.6	12.3	-1.2	-.4
Kurtosis	.4	56.6	150.6	5.6	-.3
Correlation between $\hat{X}$ and $s(\hat{X})$	.57	.69	.64	.59	.63
Proportion of correct intervals*					
Two-sided, 95.4%	22.5	51.8	95.3	94.0	95.0
Two-sided, 98.8%	22.8	52.8	97.3	97.0	98.3
One-sided, 93.3%	23.5	62.2	95.7	97.2	95.8
One-sided, 97.7%	23.5	62.2	97.2	100.0	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-96

Stratified Random Sampling of Audit Units (20 strata) Population 1
Difference Estimator n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,162	379,263	379,029	379,462	379,882
Standard deviation	485	556	1,187	1,700	2,709
Skewness	2.9	2.9	1.5	.8	.8
Kurtosis	27.8	22.5	4.9	1.4	1.9
Distribution of Z					
Number of samples with errors	264	544	600	600	600
Mean	-1.4	-4.1	-.4	-.4	-.3
Standard deviation	1.5	43.2	1.1	1.1	1.0
Skewness	.2	-13.3	.2	-.2	-.2
Kurtosis	-.8	175.6	-1.0	-.7	-.5
Correlation between $\hat{X}$ and $s(\hat{X})$	.41	.47	.56	.54	.60
Proportion of correct intervals*					
Two-sided, 95.4%	25.3	78.2	94.3	92.8	92.7
Two-sided, 98.8%	31.5	80.5	99.5	97.5	98.7
One-sided, 93.3%	44.0	90.7	97.5	97.8	96.8
One-sided, 97.7%	44.0	90.7	99.5	99.7	99.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-97

Stratified Random Sampling of Audit Units (20 strata) — Population 2
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,715	3,485,808	3,488,388	3,487,670	3,565,172
Standard deviation	5,125	12,039	30,946	39,725	86,778
Skewness	1.5	-.1	-.6	-.1	.3
Kurtosis	16.0	9.4	2.2	.7	.2
Distribution of Z					
Number of samples with errors	169	358	594	600	600
Mean	-1.0	.7	-.9	-.2	-.1
Standard deviation	2.1	1.7	3.8	1.1	1.0
Skewness	-1.8	1.9	-9.8	-.3	-.2
Kurtosis	3.8	7.2	122.5	-.2	-.0
Correlation between $\hat{X}$ and $s(\hat{X})$	.31	-.11	-.18	-.14	.26
Proportion of correct intervals*					
Two-sided, 95.4%	21.3	47.7	83.3	96.8	95.3
Two-sided, 98.8%	23.0	53.7	88.3	98.2	98.3
One-sided, 93.3%	28.2	43.8	97.7	96.3	95.0
One-sided, 97.7%	28.2	47.7	99.0	99.7	98.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-98

Stratified Random Sampling of Audit Units (20 strata)
Difference Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,810	3,485,151	3,490,075	3,489,479	3,561,386
Standard deviation	4,339	7,606	18,594	27,228	60,065
Skewness	2.2	-.6	-.4	-.2	.2
Kurtosis	19.4	3.2	1.1	.8	-.0
Distribution of Z					
Number of samples with errors	283	491	600	600	600
Mean	-1.5	1.1	-.2	-.1	-.1
Standard deviation	2.8	2.9	1.1	1.0	1.0
Skewness	-2.8	3.2	-.7	-.2	-.1
Kurtosis	9.6	14.4	.5	-.5	-.3
Correlation between $\hat{X}$ and $s(\hat{X})$	.39	-.30	-.05	-.06	.31
Proportion of correct intervals*					
Two-sided, 95.4%	39.2	64.7	95.0	97.5	96.0
Two-sided, 98.8%	39.2	66.5	97.3	99.2	99.3
One-sided, 93.3%	47.2	63.5	98.8	94.5	95.5
One-sided, 97.7%	47.2	64.7	100.0	99.5	98.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-99

Stratified Random Sampling of Audit Units (20 strata) — Population 3
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,365	13,665,642	13,648,709	13,624,464	13,504,452
Standard deviation	10,046	17,540	28,971	45,328	77,094
Skewness	-6.1	-3.8	-1.3	-1.1	-.5
Kurtosis	39.7	17.5	.8	1.0	.2
Distribution of Z					
Number of samples with errors	262	404	596	600	600
Mean	85.9	76.0	28.3	11.9	.7
Standard deviation	75.0	122.2	117.7	23.7	4.5
Skewness	.2	1.8	13.8	2.5	11.1
Kurtosis	-1.5	1.6	220.0	7.5	157.3
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.97	-.94	-.86
Proportion of correct intervals*					
Two-sided, 95.4%	7.5	15.7	46.2	60.7	88.7
Two-sided, 98.8%	7.5	15.7	46.8	63.0	91.3
One-sided, 93.3%	7.5	15.7	45.5	58.3	85.5
One-sided, 97.7%	7.5	15.7	46.2	60.7	89.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-100

Stratified Random Sampling of Audit Units (20 strata) — Population 3
Difference Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,044	13,666,162	13,647,670	13,621,811	13,512,973
Standard deviation	7,215	11,033	21,709	31,707	50,786
Skewness	-3.7	-2.4	-1.3	-.7	-.5
Kurtosis	13.7	5.5	1.8	.2	.2
Distribution of Z					
Number of samples with errors	432	546	600	600	600
Mean	163.7	107.7	9.5	3.9	.3
Standard deviation	153.4	213.1	20.1	16.7	1.2
Skewness	.3	2.5	2.2	6.1	1.5
Kurtosis	-1.6	5.0	3.8	45.3	5.4
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.99	-.94	-.93	-.87
Proportion of correct intervals*					
Two-sided, 95.4%	17.0	29.3	66.7	80.7	92.0
Two-sided, 98.8%	17.0	29.5	71.8	83.8	95.7
One-sided, 93.3%	17.0	27.8	64.0	77.8	88.0
One-sided, 97.7%	17.0	29.3	66.7	81.0	92.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-101

Stratified Random Sampling of Audit Units (20 strata) — Population 4
Difference Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,069	7,468,126	7,400,790	7,235,504	6,443,330
Standard deviation	44,304	48,862	77,258	129,973	251,371
Skewness	-1.8	-1.5	-.9	-.7	-.2
Kurtosis	2.4	1.8	.4	1.7	-.0
Distribution of Z					
Number of samples with errors	276	421	592	600	600
Mean	10.3	7.6	4.4	1.4	.1
Standard deviation	36.7	32.0	27.6	11.5	1.1
Skewness	9.8	14.2	18.0	17.6	.7
Kurtosis	105.9	239.1	376.5	358.0	.9
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.98	-.94	-.92	-.91
Proportion of correct intervals*					
Two-sided, 95.4%	31.3	41.2	71.8	85.8	93.2
Two-sided, 98.8%	31.3	43.2	74.7	89.3	98.0
One-sided, 93.3%	31.3	40.3	69.7	84.2	90.2
One-sided, 97.7%	31.3	41.2	71.8	86.2	94.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-102

Stratified Random Sampling of Audit Units (20 strata) — Population 4
Difference Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,368	7,468,193	7,403,097	7,239,300	6,445,671
Standard deviation	31,649	35,142	52,464	83,291	168,961
Skewness	-1.1	-1.0	-.7	-.4	-.2
Kurtosis	.1	.4	.4	-.1	-.1
Distribution of Z					
Number of samples with errors	415	543	600	600	600
Mean	23.6	12.5	1.5	.3	.1
Standard deviation	71.0	67.9	10.8	1.2	1.0
Skewness	8.5	13.7	20.9	1.6	.3
Kurtosis	84.8	205.6	479.5	7.3	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.97	-.93	-.95	-.93
Proportion of correct intervals*					
Two-sided, 95.4%	44.8	60.8	84.2	93.3	94.0
Two-sided, 98.8%	44.8	61.2	86.7	97.0	98.0
One-sided, 93.3%	44.5	59.3	80.2	87.5	91.2
One-sided, 97.7%	44.8	60.8	84.7	93.5	95.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-103

Stratified Random Sampling of Audit Units (15 strata) | Population 1
Ratio Estimator | n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,167	379,197	379,007	379,413	379,919
Standard deviation	867	708	1,674	2,541	4,011
Skewness	6.8	.3	2.2	1.5	1.1
Kurtosis	83.2	19.3	12.8	5.1	2.7
Distribution of Z					
Number of samples with errors	118	378	595	600	600
Mean	-.8	-2.5	-.3	-.4	-.2
Standard deviation	.9	20.3	1.0	1.1	.9
Skewness	.7	-9.5	.6	-1.0	-.1
Kurtosis	-.3	88.1	-.3	6.2	-.8
Correlation between $\hat{X}$ and $s(\hat{X})$	.63	-.04	.52	.50	.58
Proportion of correct intervals*					
Two-sided, 95.4%	18.3	55.5	97.7	94.5	97.5
Two-sided, 98.8%	19.5	58.8	98.5	98.3	99.5
One-sided, 93.3%	19.7	63.0	96.7	98.8	99.0
One-sided, 97.7%	19.7	63.0	98.3	99.8	100.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-104

Stratified Random Sampling of Audit Units (15 strata)
Ratio Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,159	379,248	379,034	379,501	379,975
Standard deviation	484	585	1,221	1,689	2,802
Skewness	4.2	4.1	1.4	1.0	.7
Kurtosis	36.6	26.2	3.9	2.8	1.3
Distribution of Z					
Number of samples with errors	245	549	600	600	600
Mean	-1.4	-2.5	-.4	-.3	-.2
Standard deviation	1.3	25.2	1.0	1.0	1.0
Skewness	.1	-16.4	.0	-.2	-.4
Kurtosis	.1	266.8	-1.0	-.6	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.59	.68	.58	.53	.60
Proportion of correct intervals*					
Two-sided, 95.4%	27.3	82.2	96.3	95.3	94.3
Two-sided, 98.8%	36.2	85.0	99.5	98.8	98.5
One-sided, 93.3%	40.8	91.5	99.0	98.8	97.5
One-sided, 97.7%	40.8	91.5	99.8	100.0	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-105

Stratified Random Sampling of Audit Units (15 strata)
Ratio Estimator

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,672	3,485,210	3,491,627	3,491,265	3,564,339
Standard deviation	6,117	10,431	26,246	34,292	84,197
Skewness	2.3	-.7	-.1	.1	.0
Kurtosis	22.7	11.4	2.0	.7	-.2
Distribution of Z					
Number of samples with errors	157	347	597	600	600
Mean	-.8	1.2	-.7	-.1	-.1
Standard deviation	1.4	3.0	3.7	1.0	.9
Skewness	-1.0	2.4	-9.0	-.2	-.4
Kurtosis	1.3	5.4	106.5	-.8	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.35	-.28	.11	.11	.39
Proportion of correct intervals*					
Two-sided, 95.4%	21.8	44.5	89.2	97.5	96.8
Two-sided, 98.8%	23.0	49.5	92.3	99.0	99.2
One-sided, 93.3%	26.2	40.5	99.2	98.3	97.3
One-sided, 97.7%	26.2	44.5	99.5	100.0	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-106

Stratified Random Sampling of Audit Units (15 strata) — Population 2
Ratio Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,487,033	3,485,776	3,490,500	3,492,067	3,569,243
Standard deviation	4,708	7,863	18,503	24,180	65,488
Skewness	1.4	-.8	.3	.2	.1
Kurtosis	7.5	3.0	.8	.4	.2
Distribution of Z					
Number of samples with errors	298	489	600	600	600
Mean	-1.2	1.3	-.1	-.0	.0
Standard deviation	2.5	3.8	1.0	1.0	1.0
Skewness	-2.7	4.6	-.6	-.2	-.4
Kurtosis	9.2	24.7	1.1	-.8	.3
Correlation between $\hat{X}$ and $s(\hat{X})$	.38	-.24	.13	.14	.44
Proportion of correct intervals*					
Two-sided, 95.4%	42.0	65.3	96.8	98.8	95.0
Two-sided, 98.8%	42.0	68.3	97.8	99.5	99.2
One-sided, 93.3%	49.7	64.3	98.2	97.2	93.5
One-sided, 97.7%	49.7	65.3	99.8	99.7	98.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-107

Stratified Random Sampling of Audit Units (15 strata) — Population 3
Ratio Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,129	13,666,800	13,648,169	13,623,314	13,513,008
Standard deviation	9,979	15,436	29,728	44,115	74,690
Skewness	-5.5	-4.8	-1.7	-1.4	-.6
Kurtosis	33.5	29.6	3.3	2.3	.3
Distribution of Z					
Number of samples with errors	227	355	595	600	600
Mean	73.9	69.8	20.2	8.7	1.0
Standard deviation	71.0	116.4	56.3	20.7	5.7
Skewness	.5	2.0	12.8	3.2	10.7
Kurtosis	-.9	2.9	224.5	13.1	122.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.97	-.96	-.95
Proportion of correct intervals*					
Two-sided, 95.4%	8.7	14.8	48.3	68.5	89.8
Two-sided, 98.8%	9.7	14.8	50.0	70.8	92.7
One-sided, 93.3%	8.7	14.8	48.0	64.5	85.5
One-sided, 97.7%	8.7	14.8	48.3	68.5	89.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-108

Stratified Random Sampling of Audit Units (15 strata)
Ratio Estimator

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,018	13,666,478	13,648,381	13,621,444	13,506,693
Standard deviation	7,168	10,723	21,385	32,152	52,673
Skewness	-4.1	-2.8	-1.1	-.8	-.2
Kurtosis	19.1	8.6	1.0	.5	-.1
Distribution of Z					
Number of samples with errors	389	525	600	600	600
Mean	140.9	100.0	11.5	3.0	.2
Standard deviation	145.5	199.2	23.0	12.5	1.1
Skewness	.5	2.5	2.3	6.0	1.8
Kurtosis	-1.0	5.2	6.3	39.3	5.8
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.99	-.95	-.94	-.93
Proportion of correct intervals*					
Two-sided, 95.4%	18.7	29.3	63.5	81.3	94.2
Two-sided, 98.8%	18.7	30.3	67.2	83.3	96.8
One-sided, 93.3%	18.7	28.5	61.7	78.3	89.7
One-sided, 97.7%	18.7	29.3	63.5	81.3	94.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-109

Stratified Random Sampling of Audit Units (15 strata) — Population 4
Ratio Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,873	7,469,167	7,401,914	7,233,187	6,430,370
Standard deviation	44,363	48,837	77,264	130,094	256,612
Skewness	-1.8	-1.6	-.9	-.4	-.0
Kurtosis	2.2	2.0	.6	.1	-.4
Distribution of Z					
Number of samples with errors	261	415	597	600	600
Mean	11.0	7.6	12.6	.8	.1
Standard deviation	30.2	24.6	238.9	3.1	1.0
Skewness	5.9	8.4	24.3	5.3	.7
Kurtosis	39.7	87.6	590.1	42.0	.9
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.98	-.95	-.94	-.96
Proportion of correct intervals*					
Two-sided, 95.4%	29.5	41.2	74.0	88.3	96.0
Two-sided, 98.8%	29.5	41.8	76.3	90.3	97.8
One-sided, 93.3%	29.5	39.7	70.8	84.3	91.2
One-sided, 97.7%	29.5	41.2	74.0	88.3	96.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-110

Stratified Random Sampling of Audit Units (15 strata) — Population 4
Ratio Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,480,429	7,469,885	7,402,129	7,241,942	6,443,243
Standard deviation	30,352	33,689	52,272	86,419	170,362
Skewness	-1.2	-1.0	-.4	-.2	.2
Kurtosis	.4	.3	-.6	-.1	.2
Distribution of Z					
Number of samples with errors	399	548	600	600	600
Mean	22.0	11.7	1.1	.3	.1
Standard deviation	63.5	47.0	4.7	1.4	1.0
Skewness	6.3	10.5	10.5	2.0	.9
Kurtosis	44.9	132.2	160.2	6.9	2.6
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.97	-.94	-.95	-.96
Proportion of correct intervals*					
Two-sided, 95.4%	43.5	60.5	82.8	92.3	95.2
Two-sided, 98.8%	43.5	62.2	86.7	94.5	98.2
One-sided, 93.3%	42.3	58.5	78.2	85.8	91.5
One-sided, 97.7%	43.5	60.5	82.8	92.5	95.7

*Interval is considered incorrect if estimated standard error is zero.

Table A-111

Stratified Random Sampling of Audit Units (20 strata) — Population 1
Ratio Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,158	379,278	379,172	379,638	380,330
Standard deviation	771	825	1,996	2,765	4,173
Skewness	7.3	6.2	2.6	2.1	.9
Kurtosis	99.5	71.9	12.2	9.5	1.7
Distribution of Z					
Number of samples with errors	141	373	591	600	600
Mean	-1.0	-5.0	1.2	-.3	-.2
Standard deviation	.9	34.0	17.9	1.1	.9
Skewness	1.0	-7.7	12.3	-1.2	-.4
Kurtosis	.2	56.8	152.1	5.9	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.55	.68	.64	.59	.62
Proportion of correct intervals*					
Two-sided, 95.4%	22.8	52.2	97.0	96.2	97.8
Two-sided, 98.8%	23.3	53.3	97.5	98.8	99.2
One-sided, 93.3%	23.5	62.2	97.0	99.8	98.5
One-sided, 97.7%	23.5	62.2	97.3	100.0	100.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-112

Stratified Random Sampling of Audit Units (20 strata)
Ratio Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264	379,090	379,547	379,921
Distribution of $\hat{X}$					
Mean	379,162	379,263	379,029	379,462	379,882
Standard deviation	485	557	1,187	1,701	2,707
Skewness	2.9	3.0	1.5	.8	.8
Kurtosis	28.0	22.7	4.9	1.4	1.9
Distribution of Z					
Number of samples with errors	264	544	600	600	600
Mean	-1.4	-3.9	-.4	-.4	-.2
Standard deviation	1.5	41.9	1.0	1.0	1.0
Skewness	.2	-13.3	.2	-.2	-.2
Kurtosis	-.8	175.6	-1.0	-.7	-.5
Correlation between $\hat{X}$ and $s(\hat{X})$	.41	.47	.56	.55	.60
Proportion of correct intervals*					
Two-sided, 95.4%	26.2	78.5	96.3	95.3	95.7
Two-sided, 98.8%	31.7	81.5	100.0	97.8	99.3
One-sided, 93.3%	44.0	90.7	97.5	99.2	98.2
One-sided, 97.7%	44.0	90.7	99.7	100.0	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-113

Stratified Random Sampling of Audit Units (20 strata) | Population 2
Ratio Estimator | n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,784	3,485,883	3,488,463	3,487,752	3,565,252
Standard deviation	5,124	12,028	30,963	39,763	86,764
Skewness	1.4	-.1	-.6	-.1	.3
Kurtosis	16.0	9.3	2.2	.7	.2
Distribution of Z					
Number of samples with errors	169	358	594	600	600
Mean	-.8	.6	-.8	-.2	-.1
Standard deviation	1.7	1.5	3.3	1.0	.9
Skewness	-1.8	2.1	-9.6	-.3	-.2
Kurtosis	3.8	9.7	117.9	-.1	-.2
Correlation between $\hat{X}$ and $s(\hat{X})$	.32	-.07	-.26	-.19	.26
Proportion of correct intervals*					
Two-sided, 95.4%	23.0	49.8	86.8	98.0	97.8
Two-sided, 98.8%	24.7	54.2	91.0	98.7	99.5
One-sided, 93.3%	28.2	44.8	98.7	99.0	97.7
One-sided, 97.7%	28.2	49.8	99.0	100.0	99.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-114

Stratified Random Sampling of Audit Units (20 strata)
Ratio Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value	3,487,012	3,485,576	3,490,751	3,490,954	3,564,610
Distribution of $\hat{X}$					
Mean	3,486,881	3,485,221	3,490,148	3,489,547	3,561,456
Standard deviation	4,335	7,608	18,602	27,243	60,025
Skewness	2.2	-.6	-.4	-.2	.2
Kurtosis	19.2	3.2	1.1	.8	-.0
Distribution of Z					
Number of samples with errors	283	491	600	600	600
Mean	-1.2	1.2	-.2	-.1	-.1
Standard deviation	2.4	2.9	1.1	1.0	.9
Skewness	-2.7	3.4	-.7	-.2	-.1
Kurtosis	9.5	16.0	.4	-.5	-.4
Correlation between $\hat{X}$ and $s(\hat{X})$	.40	-.29	-.06	-.06	.32
Proportion of correct intervals*					
Two-sided, 95.4%	39.2	65.0	96.0	98.0	97.2
Two-sided, 98.8%	39.2	66.8	97.8	99.3	99.7
One-sided, 93.3%	47.2	63.7	99.2	95.5	96.2
One-sided, 97.7%	47.2	65.0	100.0	99.8	99.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-115

Stratified Random Sampling of Audit Units (20 strata) — Population 3
Ratio Estimator — n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,370	13,665,638	13,648,694	13,624,433	13,504,409
Standard deviation	10,028	17,567	29,001	45,410	77,176
Skewness	-6.1	-3.8	-1.3	-1.1	-.5
Kurtosis	39.8	17.7	.8	1.0	.2
Distribution of Z					
Number of samples with errors	262	404	596	600	600
Mean	77.8	68.9	25.6	10.7	.6
Standard deviation	67.9	110.7	108.3	21.4	4.0
Skewness	.2	1.8	14.0	2.5	11.7
Kurtosis	-1.6	1.6	226.2	7.4	174.7
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00	-.99	-.98	-.96	-.94
Proportion of correct intervals*					
Two-sided, 95.4%	7.5	15.7	46.7	62.3	91.3
Two-sided, 98.8%	7.5	15.7	46.8	64.7	93.0
One-sided, 93.3%	7.5	15.7	46.0	59.7	88.5
One-sided, 97.7%	7.5	15.7	46.7	62.3	91.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-116

Stratified Random Sampling of Audit Units (20 strata) — Population 3

Ratio Estimator — n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964	13,666,230	13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,045	13,666,168	13,647,678	13,621,829	13,512,976
Standard deviation	7,207	11,009	21,688	31,681	50,800
Skewness	-3.7	-2.4	-1.3	-.7	-.5
Kurtosis	13.6	5.4	1.8	.2	.2
Distribution of Z					
Number of samples with errors	432	546	600	600	600
Mean	156.1	102.6	9.0	3.7	.3
Standard deviation	146.1	202.7	19.1	15.8	1.1
Skewness	.3	2.5	2.2	6.1	1.7
Kurtosis	-1.6	4.9	3.9	45.3	6.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-.99	-.99	-.95	-.94	-.93
Proportion of correct intervals*					
Two-sided, 95.4%	17.0	29.3	67.5	81.8	94.3
Two-sided, 98.8%	17.0	29.5	72.0	84.5	97.0
One-sided, 93.3%	17.0	27.8	64.5	78.0	90.0
One-sided, 97.7%	17.0	29.3	67.5	81.8	94.3

*Interval is considered incorrect if estimated standard error is zero.

Table A-117

Stratified Random Sampling of Audit Units (20 strata)
Ratio Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,085	7,468,128	7,400,762	7,235,439	6,443,132
Standard deviation	44,276	48,865	77,288	129,981	251,129
Skewness	-1.8	-1.5	-.9	-.7	-.2
Kurtosis	2.4	1.8	.4	1.7	-.0
Distribution of Z					
Number of samples with errors	276	421	592	600	600
Mean	9.3	6.9	4.0	1.3	.1
Standard deviation	33.4	28.9	24.5	10.4	.9
Skewness	9.9	14.3	18.0	17.8	.6
Kurtosis	106.3	242.6	375.1	364.3	.8
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.97	-.94	-.93	-.95
Proportion of correct intervals*					
Two-sided, 95.4%	31.3	42.8	73.0	87.7	95.7
Two-sided, 98.8%	31.3	43.8	76.2	90.5	98.5
One-sided, 93.3%	31.3	40.5	70.8	85.0	91.0
One-sided, 97.7%	31.3	42.8	73.0	87.8	96.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-118

Stratified Random Sampling of Audit Units (20 strata)
Ratio Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146	7,468,741	7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,375	7,468,202	7,403,127	7,239,357	6,445,788
Standard deviation	31,641	35,128	52,399	83,155	168,741
Skewness	-1.1	-1.0	-.7	-.4	-.2
Kurtosis	.1	.4	.4	-.1	-.1
Distribution of Z					
Number of samples with errors	415	543	600	600	600
Mean	22.5	12.0	1.4	.2	.1
Standard deviation	67.6	64.7	10.1	1.1	1.0
Skewness	8.5	13.7	20.9	1.7	.3
Kurtosis	84.7	205.5	477.9	7.4	.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98	-.96	-.94	-.95	-.96
Proportion of correct intervals*					
Two-sided, 95.4%	44.8	61.0	85.2	94.2	95.2
Two-sided, 98.8%	44.8	61.3	87.8	97.3	99.0
One-sided, 93.3%	44.8	59.5	80.8	88.7	91.8
One-sided, 97.7%	44.8	61.0	85.2	94.3	95.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-119

Dollar Unit Sampling
CAV Bound

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total error amount	2,536		23,671	48,704	161,661
Distribution of 95% UCL					
Mean	451,077		726,868	867,349	1,305,470
Standard deviation	66,956		113,289	119,093	132,954
Skewness	1.1		.3	.7	.7
Kurtosis	.3		.6	1.9	.3
Maximum	699,747		1,112,849	1,405,424	1,765,934
Minimum	403,525		403,525	569,258	1,013,245
Proportion of correct limits	100.0		100.0	100.0	100.0

Table A-120

Dollar Unit Sampling
CAV Bound

Population 3
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total error amount	2,536		23,671	48,704	161,661
Distribution of 95% UCL					
Mean	254,557		476,093	590,621	937,440
Standard deviation	43,489		66,085	74,232	86,642
Skewness	.6		.2	.5	.4
Kurtosis	.1		.6	.4	-.3
Maximum	432,383		746,665	864,404	1,204,395
Minimum	203,270		252,966	388,438	733,551
Proportion of correct limits	100.0		100.0	100.0	100.0

Table A-121

Dollar Unit Sampling
CAV Bound

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total error amount	24,811		100,607	265,678	1,060,586
Distribution of 95% UCL					
Mean	276,366		477,350	743,575	1,681,046
Standard deviation	70,602		113,719	162,416	263,126
Skewness	1.2		.3	.4	.2
Kurtosis	.7		-.3	.2	-.1
Maximum	568,124		809,063	1,280,875	2,495,349
Minimum	221,455		221,455	362,168	949,994
Proportion of correct limits	100.0		100.0	100.0	99.3

Table A-122

Dollar Unit Sampling
CAV Bound

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total error amount	24,811		100,607	265,678	1,060,586
Distribution of 95% UCL					
Mean	163,850		336,893	575,236	1,489,494
Standard deviation	48,425		76,169	103,470	177,834
Skewness	1.0		.5	.2	.3
Kurtosis	1.0		.1	.3	.1
Maximum	368,614		591,028	1,030,524	2,072,322
Minimum	111,554		160,722	263,836	983,331
Proportion of correct limits	100.0		100.0	99.8	99.5

Table A-123

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 1
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,173	379,283		379,593	379,874
Standard deviation	789	813		2,308	3,640
Skewness	6.4	5.3		1.7	1.3
Kurtosis	60.4	43.0		5.6	5.2
Distribution of Z					
Number of samples with errors	121	365		600	600
Mean	-1.1	-7.2		-.4	-.3
Standard deviation	1.2	37.0		1.2	1.1
Skewness	.2	-5.7		-.8	-.5
Kurtosis	.1	31.1		1.6	-.1
Correlation between $\hat{X}$ and $s(\hat{X})$	.70	.73		.64	.65
Proportion of correct intervals*					
Two-sided, 95.4%	17.5	49.7		90.3	92.2
Two-sided, 98.8%	18.2	53.7		96.3	97.2
One-sided, 93.3%	20.2	60.3		98.7	98.2
One-sided, 97.7%	20.2	60.7		99.7	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-124

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 1
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	379,181	379,264		379,547	379,921
Distribution of $\hat{X}$					
Mean	379,172	379,264		379,543	379,837
Standard deviation	528	561		1,761	2,563
Skewness	4.1	3.1		1.3	.7
Kurtosis	26.8	17.8		3.3	1.2
Distribution of Z					
Number of samples with errors	234	525		600	600
Mean	-1.5	-4.6		-.3	-.3
Standard deviation	1.9	39.6		1.1	1.1
Skewness	-1.0	-11.2		-.2	-.3
Kurtosis	1.5	124.3		-.8	-.7
Correlation between $\hat{X}$ and $s(\hat{X})$	.64	.64		.67	.62
Proportion of correct intervals*					
Two-sided, 95.4%	27.5	71.7		92.7	93.0
Two-sided, 98.8%	33.3	75.0		98.2	98.5
One-sided, 93.3%	39.0	87.0		97.7	97.8
One-sided, 97.7%	39.0	87.5		99.5	99.8

*Interval is considered incorrect if estimated standard error is zero.

Table A-125

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 2
n = 100

	Population Error Percentage				
	.5	1	5	10	70
Total audit value			3,490,751	3,490,954	
Distribution of $\hat{X}$					
Mean			3,490,795	3,494,217	
Standard deviation			33,228	50,774	
Skewness			3.4	4.6	
Kurtosis			34.8	44.2	
Distribution of Z					
Number of samples with errors			595	600	
Mean			-1.3	-.2	
Standard deviation			5.0	1.1	
Skewness			-7.7	-.5	
Kurtosis			68.2	-.3	
Correlation between $\hat{X}$ and $s(\hat{X})$			.53	.70	
Proportion of correct intervals*					
Two-sided, 95.4%			80.2	94.5	
Two-sided, 98.8%			85.2	97.7	
One-sided, 93.3%			98.7	98.7	
One-sided, 97.7%			99.2	100.0	

*Interval is considered incorrect if estimated standard error is zero.

Table A-126

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 2
n = 200

	Population Error Percentage				
	.5	1	5	10	70
Total audit value			3,490,751	3,490,954	
Distribution of $\hat{X}$					
Mean			3,490,364	3,493,185	
Standard deviation			22,024	32,002	
Skewness			2.0	2.3	
Kurtosis			14.6	16.3	
Distribution of Z					
Number of samples with errors			600	600	
Mean			-.4	-.1	
Standard deviation			1.7	1.0	
Skewness			-6.5	-.3	
Kurtosis			91.6	-.6	
Correlation between $\hat{X}$ and $s(\hat{X})$			.49	.63	
Proportion of correct intervals*					
Two-sided, 95.4%			91.5	97.3	
Two-sided, 98.8%			93.7	99.5	
One-sided, 93.3%			98.0	96.7	
One-sided, 97.7%			100.0	100.0	

*Interval is considered incorrect if estimated standard error is zero.

Table A-127

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 3
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964		13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,669,037		13,646,511	13,623,357	13,504,032
Standard deviation	14,272		46,638	68,871	123,075
Skewness	-7.3		-2.3	-2.1	-.8
Kurtosis	55.8		5.4	6.2	.4
Distribution of Z					
Number of samples with errors	227		596	600	600
Mean	109.4		32.0	26.5	8.4
Standard deviation	80.4		105.3	36.9	24.8
Skewness	-.3		14.4	2.5	3.3
Kurtosis	-1.8		227.1	13.0	10.1
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00		-.98	-.96	-.94
Proportion of correct intervals*					
Two-sided, 95.4%	5.2		31.5	44.8	77.0
Two-sided, 98.8%	5.2		36.2	46.8	77.5
One-sided, 93.3%	5.2		31.5	44.2	76.8
One-sided, 97.7%	5.2		31.5	44.8	77.0

*Interval is considered incorrect if estimated standard error is zero.

Table A-128

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 3
$n = 200$

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	13,668,964		13,647,829	13,622,796	13,509,839
Distribution of $\hat{X}$					
Mean	13,668,644		13,646,367	13,618,858	13,504,206
Standard deviation	11,061		33,083	48,769	83,359
Skewness	-4.6		-1.6	-1.0	-.4
Kurtosis	20.9		2.4	.8	-.3
Distribution of Z					
Number of samples with errors	413		600	600	600
Mean	189.9		23.4	15.4	1.9
Standard deviation	156.0		41.5	30.3	12.8
Skewness	.0		8.3	1.9	9.4
Kurtosis	-1.8		117.4	2.6	93.0
Correlation between $\hat{X}$ and $s(\hat{X})$	-1.00		-.97	-.95	-.94
Proportion of correct intervals*					
Two-sided, 95.4%	10.7		47.0	62.7	88.2
Two-sided, 98.8%	10.7		49.3	65.0	89.5
One-sided, 93.3%	10.7		46.7	62.0	85.3
One-sided, 97.7%	10.7		47.0	62.7	88.2

*Interval is considered incorrect if estimated standard error is zero.

Table A-129

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 4
n = 100

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146		7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,478,910		7,401,538	7,231,457	6,453,326
Standard deviation	41,018		76,494	132,135	253,581
Skewness	-1.6		-.6	-.6	-.2
Kurtosis	2.1		-.3	.4	-.0
Distribution of Z					
Number of samples with errors	286		595	600	600
Mean	10.4		4.4	.8	.2
Standard deviation	15.0		11.9	4.1	1.1
Skewness	.7		5.5	6.6	.5
Kurtosis	-1.5		41.2	55.0	.6
Correlation between $\hat{X}$ and $s(\hat{X})$	-.98		-.95	-.95	-.99
Proportion of correct intervals*					
Two-sided, 95.4%	30.7		69.7	86.5	94.5
Two-sided, 98.8%	30.7		69.7	95.7	97.2
One-sided, 93.3%	30.7		69.7	83.0	89.7
One-sided, 97.7%	30.7		69.7	87.0	95.5

*Interval is considered incorrect if estimated standard error is zero.

Table A-130

Dollar Unit Sampling
Mean-Per-Unit Estimator

Population 4
n = 200

	Population Error Percentage				
	.5	1	5	10	30
Total audit value	7,478,146		7,402,350	7,237,279	6,442,371
Distribution of $\hat{X}$					
Mean	7,479,345		7,404,301	7,237,593	6,438,854
Standard deviation	29,817		57,385	89,568	175,986
Skewness	-1.4		-.7	-.3	-.3
Kurtosis	2.3		.2	.2	.1
Distribution of Z					
Number of samples with errors	431		600	600	600
Mean	17.9		1.7	.3	.1
Standard deviation	28.6		5.6	2.0	1.0
Skewness	1.1		5.1	9.6	.2
Kurtosis	-.5		35.7	130.9	.2
Correlation between $\hat{X}$ and $s(\hat{X})$	-.95		-.95	-.98	-.99
Proportion of correct intervals*					
Two-sided, 95.4%	46.0		89.8	94.3	95.2
Two-sided, 98.8%	46.2		90.0	95.3	98.2
One-sided, 93.3%	45.7		88.0	87.5	92.5
One-sided, 97.7%	46.0		90.0	94.5	96.8

*Interval is considered incorrect if estimated standard error is zero.